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RESEARCH ARTICLE

Knowledge of Mathematics Content and its Relation to the Mathematical pedagogical content knowledge for Secondary School Teachers

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ABSTRACT

This research aims to identify the degree of mathematics content knowledge (MCK) among secondary school mathematics teachers and its relationship to mathematical pedagogical content knowledge (MPCK). To achieve the research objective, the researcher prepared two instruments: a mathematics content knowledge test and a mathematical pedagogical content knowledge scale for secondary mathematics teachers. Then, the validity and reliability of the instruments were checked. The researcher followed a descriptive-analytical approach. The sample comprised 49 female mathematics teachers, representing 30% of the study's population (166). The results of the research were: (1) The means of the MCK test criteria ranged between 0.674 and 0.404, with an overall mean of 0.555, a percentage of 55.5%, and a standard deviation of 0.468, which indicated an intermediate level of MCK among secondary school mathematics teachers in Dammam and Khobar; (2) The means of the MPCK scale criteria ranged between 3.326 and 2.796, with an overall mean of 2.974, a percentage of 99.8%, and a standard deviation of 0.918, which indicates an intermediate level of the MPCK among secondary school mathematics teachers in the named cities. (3) There is no statistically significant relationship between the MCK and the MPCK associated with it among secondary school mathematics teachers. In light of the results obtained, the research recommended the need to consider professional development in the MPCK based on teachers' professional standards, the diversity in various professional development activities, the activation of professional learning societies among teachers, and the need to reconsider the programs for preparing the necessary mathematics teachers professionally and educationally and conduct more studies on the knowledge of mathematics content and the mathematical pedagogical content knowledge.

INTRODUCTION

The Kingdom pays great importance to human capital, realizing that it is the actual investment, sustainable wealth, and way to achieve the goals of Vision 2030, which includes strategic plans and development steps in all fields, locally and globally. These include the National Transformation Programme, which affects the education sector and provides it with many initiatives, i.e., a "comprehensive framework for teachers' continuous professional development, the development of a national strategy to raise the level of the teaching profession by raising teachers' professional level, and the development of the system surrounding the profession." Many studies have emphasized the need to build Professional development programs concerned with PCK, which include specialized scientific aspects and how to present them to the learner in light of modern teaching and learning theories (Al-Shamrani, 2017).

In 2000, a team from the Centre of Teaching and Learning Mathematics at Michigan University investigated how Shulman's ideas could be applied to mathematics education through two projects: The Mathematics Teaching and Learning to Teach (MTLT) project and Learning Mathematics For Teaching (LMT); to develop a practice-based theory of the concept of Mathematical Knowledge needed for teaching mathematics, focusing on the type of professional knowledge needed for teaching mathematics to support their students' Education. Michigan's team identified the Mathematical Knowledge and skills that the teachers need to teach mathematics more effectively. They divided them into two main parts: the first is subject matter knowledge (SMK), which includes three sub-domains: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). The second is pedagogical content knowledge (PCK), which includes three sub-domains: Knowledge of content and students (KCS), Knowledge of content and teaching (KCT), and Knowledge of content and curriculum (KCC) (Loewenberg Ball et al., 2008).

Concerning MCK, the National Council of Teachers of Mathematics (NCTM) in the United States of America (1991) called for teachers of mathematics to have deep specialized knowledge represented in concepts, procedural knowledge, mathematical algorithms, and their representations, linking them together, all kinds of mathematical proof, problem-solving, mathematical communication, knowledge of school mathematics, methods of teaching and learning, its relationship to other subjects, and its applications in society.

On the other hand, the PCK is an essential element of the teacher's knowledge needed for teaching, which helps him make the appropriate decisions for high-quality teaching. The researchers agreed on integrating pedagogical Knowledge and specialized mathematics content Knowledge. This is represented by the PCK (Maryono et al., 2017). Teachers' mastery of the specialized content and high mathematical skills must be improved for classroom success. Instead, it is necessary to have the knowledge and skills needed to present that specialized content in a way that the students can understand and make them able to use it in real-life situations outside the classroom (El-Mutreb et al., 2017). Therefore, the teacher must understand the content he teaches.

Many studies have been conducted to reveal the extent to which mathematics teachers in the primary stage can acquire mathematical Knowledge and pedagogical Knowledge. For example, El-Mutreb and Al-Salouli (2015), Al-Salouli (2016), and El-Mutreb et al. (2017) reached a low level of mathematical knowledge and their cognitive depth of teaching different topics, such as geometry, numbers with arithmetic operations, using decimal numbers or fractions, and other topics that appear in primary school mathematics. There is also a low level in many aspects, such as methods of students' evaluation, judging the validity of their alternative methods in the solution, and the possibility of generalization.

Some studies have also shown that there is a disparity in the levels of mastery of pedagogical knowledge; it is between a high level of Proficiency (Hussein, 2013) or an intermediate level of Proficiency in the specialized and pedagogical aspects (Al-Sir, 2016 A), an intermediate level (Al-Sir, 2016 b), or a low level (Abdel-Alal, 2017).

Accordingly, MCK and the PCK associated with it are two integral parts of effective mathematics teaching. Besides knowing the mathematics content itself, there is always a need for MPCK to build mathematically

solid concepts in students' minds. When the teacher of mathematics presents the lesson well, he indeed links his knowledge of the topic (i.e., his knowledge of what he is teaching) and his pedagogical knowledge of teaching and students (i.e., his knowledge of how to teach) (Shulman, 1986).

Therefore, studies have emphasized the importance of the relationship between MCK and MPCK and that there is a positive correlation between the teacher's knowledge of the content and the knowledge required for teaching. Teachers with high knowledge of mathematics tend to use problem-solving as a teaching strategy, while teachers with insufficient knowledge tend to have limited use of problem-solving strategies(Kanyongo and Brown, 2013).

Statement of the problem

Studies have shown Mathematics content knowledge (MCK) is a pre-requisite of pedagogical content knowledge (PCK) (Agathangelou and Charalambous, 2021; Charalambous et al., 2020). Also, Studies have shown a low level of teachers' MCK and its relationship to PCK (Moh'd et al., 2021; El-Mutreb and Al-Salouli, 2015; Al-Salouli, 2016; El-Mutreb et al., 2017). Because of the great responsibility of teachers and their possession of knowledge of the content of mathematics and educational knowledge in achieving the quality of teaching and the results of mathematics learning for students and making the knowledge and skills understandable to them (Yang and Kaiser, 2022), it is necessary to do their best in preparing it and assessing the extent to which it acquires the mathematical knowledge needed for teaching mathematics (Hussein, 2013). In addition, there is an interest in investigating "Content Knowledge" and "PCK" in line with the increasing interest in professional development research for mathematics teachers (Pérez-Montilla and Arnal-Palacián, 2023; Twaddle and Smith, 2023; Minken et al., 2021). Despite the diversity of the global studies on teacher knowledge (Yang and Kaiser, 2022; Copur-Gencturk and Tolar, 2022; Oattes et al., 2022), the studies in the Kingdom of Saudi Arabia did not address MCK and its relationship to PCK among teachers. Therefore, this Research aims to determine the level of MCK among secondary school teachers and its relationship to the level of MPCK.

The main issue of the study is "What is the level of MCK and its relationship to PCK among secondary school mathematics teachers?"

This question branches out into the following questions:

- What is the level of MCK among secondary school mathematics teachers?
- What is the level of MPCK among secondary school mathematics teachers?
- What is the relationship between MCK and MPCK among secondary school mathematics teachers?

Objectives of the research

The current Research seeks to achieve the following objectives:

- Detecting the level of MCK among secondary school mathematics teachers
- Detecting the level of MPCK among secondary school mathematics teachers
- Determining the relationship between the level of MCK and MPCK among secondary school mathematics teachers.

The theoretical and practical significance of the research

Providing researchers with a tool for measuring the level of MCK and a tool for measuring MPCK among secondary school teachers. Also, providing future visions for educational decision-makers about the importance of evaluating teachers' MCK and evaluating teaching performance.

Presenting some recommendations and suggestions for further Research and studies to develop mathematics teacher preparation programs before service and to develop them professionally during service. Providing those interested in Education in the Kingdom of Saudi Arabia with a level of MCK and its relationship to the MPCK of secondary school teachers, which contributes to designing development projects; finally, developing training programs for educational supervisors according to the actual needs of mathematics teachers to achieve the desired goals.

LITERATURE REVIEW

Mathematics Content Knowledge (MCK)

Shulman (1986) paid attention to teacher preparation programs and emphasized that these programs should not focus only on developing pedagogical skills and knowledge of teaching methods, nor on

programs for developing specialized skills and content Knowledge, but rather on knowledge resulting from their intersection. The knowledge needed for teaching should combine the two fields of knowledge, known as PCK.

It is also the teacher's understanding of how to help students understand a particular topic. It is transforming the academic content into something learnable for a group of students studying at a specific school (Siam and Al-Astal, 2014: 25). The PCK is a unique mixture of two basic concepts: pedagogical Knowledge (PK) and Content Knowledge (CK) (Abdel-Alal, 2017; Othman and Al-Abed, 2018).

Shulman (1986) presented a model of a teacher's knowledge consisting of seven components: (1) Subject Matter Knowledge (SMK); (2) Curriculum Knowledge (CK); (3) Pedagogical Content Knowledge (PCK); (4) General Pedagogical Knowledge (GPK); (5) Learners and Learning Knowledge; (6) Knowledge of Contexts (KC); (7) Knowledge of Educational Philosophies, Goals, and Objectives (Petrou and Goulding, 2011).

In light of this, two areas of teacher knowledge can be identified: the Public domain, which includes the last four categories, and the specific domain, which is the field that Shulman focused on in his research and includes the first three categories: SMK, CK, and PCK. These categories describe the content dimensions of teacher knowledge and form "the missing model," as Shulman called it. They are as follows (Abdel-Alal, 2017; Petrou and Goulding, 2011; Shulman, 1986):

- 1. Subject Matter Knowledge (SMK) is the knowledge of the subject and its organizational structures. It is defined as the quantity and organization of knowledge in the teacher's mind.
- 2. Curriculum Knowledge (CK): This is the knowledge of the available educational materials, Such as curricula and textbooks. Shulman referred to two dimensions of methodological knowledge.
- 3. Pedagogical Content Knowledge (PCK) is a unique combination of knowledge that goes beyond knowing the academic content to knowing the most valuable teaching methods.

Mathematical Knowledge for Teaching (MKT)

The Shulman-advocated theoretical framework has led to a qualitative change in the teaching process. It has refocused researchers' attention on the content dimensions of the teacher's Knowledge (SMK and PCK), where the main focus was on teaching principles (Copur-Gencturk and Tolar, 2022; Oattes et al., 2022; Minken et al., 2021).

Loewenberg Ball et al. (2008) suggested that MKT is one of the most critical concepts for MCK in its teaching context, and it is divided into SMK and PCK. Loewenberg Ball et al. (2008) also defined knowledge of content and students (KCS), Knowledge of content and teaching (KCT), and Knowledge of content and curriculum (KCC) to be included in PCK (Ruhşen Aldemir et al., 2023; Dewi et al., 2020; Busi and Jacobbe, 2018; Khakbaz, 2016).

Kula Ünver (2018), Deniz Yilmaz and Küçük Demir (2021) also proposed the Knowledge Quartet (KQ) model, which aims to determine the different aspects of a teacher's knowledge, whether the SMK or the PCK, that impact the effectiveness of teaching. It is a framework that connects theory and practices practice by studying teaching performance. These four components are determined by the following:

Foundations: These refer to the teacher's theoretical background and beliefs related to the teacher's knowledge and appear in planning and teaching. This background consists of Knowledge and Understanding of the mathematics content itself, mathematics content beliefs, and knowledge of mathematics teaching methods.

Transformation: The teacher can transform CK into other forms, Such as representations, explanations, illustrations, similes, and examples, to help students form the concept.

Connection: Connection means the coherence of the mathematical content in the teacher's mind, the ability to sequence teaching, the awareness of the cognitive requirements of the subjects, and the different tasks.

Contingency: Contingency means the teacher's ability to anticipate more students' responses to deal with it. The greater the teacher's knowledge, the greater his anticipation and preparations, and accordingly, the fewer surprises when teaching.

Mathematical Content Knowledge (MCK)

There are different definitions of MCK. Shulman (1986, 9) defined it as the amount and organization of knowledge in the teacher's mind. Campbell et al. (2014) defined it as knowledge related to the content of school mathematics, and it includes knowledge of facts, mathematical procedures, concepts, and generalizations related to school mathematics curricula.

Dreher et al. (2018) indicated the need for mathematics teachers to have a particular type of CK called school-related content knowledge (SRCK), which describes the correlation between the academic mathematics teachers teach during their undergraduate studies and the school mathematics they teach. According to Cohen and Ball (2000), mathematics teaching and learning result from continuous interactions between the teacher, the students, and the mathematics, so concentration should not be only on each element separately but also on the interactions between teachers and students around the content.

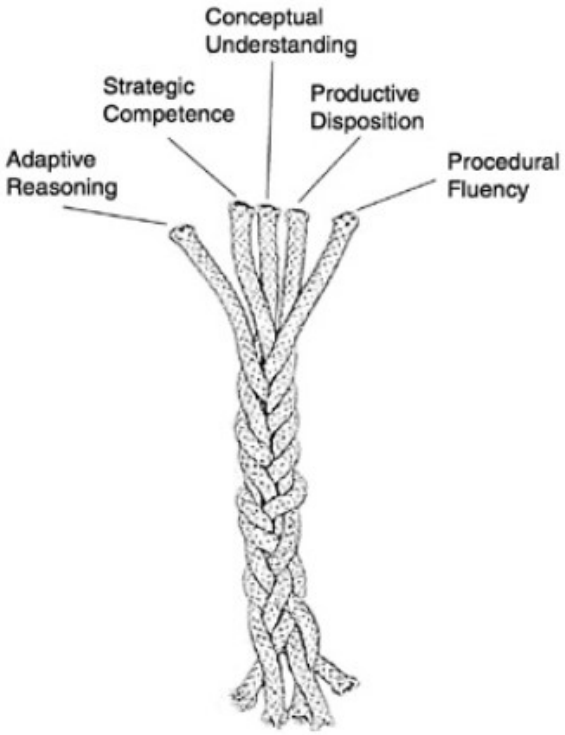

Figure 1: The intertwined strands of proficiency (National Research Council (NRC), 2001)

One of the general trends in learning mathematics school is to develop thinking skills and achieve mathematical proficiency among students. The Mathematical Proficiency includes all aspects of experience, competence, and mathematical knowledge by focusing on five correlated and integrated components; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council (NRC), 2001).

Conceptual Understanding means Understanding mathematical concepts, operations, and relationships, in which students learn new mathematical ideas by linking them to their previous mathematical experiences and knowledge (National Research Council (NRC), 2001).

Procedural fluency is a skill in executing procedures flexibly, accurately, efficiently, and appropriately, which gives the student speed and efficiency in performing mathematical operations, whether mental or written. They are related to the tasks of daily life (National Research Council (NRC), 2001).

Strategic Competence means the ability to formulate, represent, and solve mathematical problems. The learner is faced with a complex situation in real life, which requires dealing with it with problem-solving strategies and rephrasing it to deal with it mathematically, symbolically, verbally, or graphically, and constructing a mental image of its essential components, mathematical representations of the problem, discovering the mathematical relations within it and creating new solutions (Al-Moatham and Al-Menoufy, 2014).

Adaptive reasoning refers to the ability to think logically and to interpret and justify a situation appropriately (National Research Council (NRC), 2001).

The three components of Mathematical Proficiency, "Conceptual Understanding, Procedural Fluency, and Strategic Competence," are integrated to form the ability for adaptive reasoning (Al-Moatham and Al-Menoufy, 2014).

A productive disposition is a natural tendency to see mathematics as logical, Valuable, and worthwhile. It does not depend on ability; the result will be a student proficient in mathematics with Understanding, procedural fluency, strategic competence, and adaptive reasoning abilities (National Research Council (NRC), 2001).

Many studies have been conducted on MCK, such as Al-Hlaisi and Al-Salouli (2016), which concluded that the level of mathematics teachers' teaching practices of conceptual knowledge was intermediate, while the level of teaching practices of their procedural knowledge was high. The level of balance in their teaching practices of conceptual and procedural knowledge aspects was intermediate. Lowrie and Jorgensen (2016) found that three factors determine teachers' mathematical competence and affect their point of view towards mathematics teaching: inquiry-based teaching, how to acquire mathematical knowledge, and applying mathematics.

Dreher et al. (2018) emphasized the importance of providing SRCK, which includes teacher Knowledge about the structure of school mathematics curricula and the interrelationships between school and academic mathematics. Cho and Tee (2018) emphasized the importance of basic and advanced MCK. Another research showed that the level of teachers' conceptual Knowledge, procedural Knowledge, and mathematical thinking was deficient and that there were statistically significant differences between teachers' performance in the conceptual Knowledge test and their performance in the procedural Knowledge test in favor of procedural knowledge.

Al-Shammari et al. (2019) showed that mathematics teachers' practice of mathematical Proficiency came in the following order: conceptual Understanding to an intermediate degree, while procedural fluency, strategic competence, adaptive reasoning, and productive desire were weak, and the degree of mathematics teachers' practice of mathematical Proficiency was weak. Al-Khazim (2020) showed high epistemological perceptions of conceptual knowledge (the mathematics teacher's opinions and ideas about mathematical, conceptual knowledge, structure, and how to form, acquire, teach, learn, and evaluate it). Ebio (2022) Showed the existence of difficulties in the mathematics content areas related to numbers, numerical sense, measurement, patterns, algebra, geometry, and probability and statistics content areas of mathematics due to the lack of skills related to translating mathematical sentences into mathematical equations, poor reading comprehension that leads to incorrect application of mathematical formulas, and a lack of mastery of mathematical concepts.

Pedagogical Content Knowledge (PCK)

It is defined as the knowledge that makes teachers, not scientists, as teachers differ from scientists, not in the quality and quantity of their content knowledge but in how that content is organized (Loewenberg Ball et al., 2008). is used to express teachers' interpretations of mathematics knowledge to support student learning (Deniz Yilmaz and Küçük Demir, 2021; Dewi et al., 2020; Gess-Newsome et al., 2019). It is also defined as the teacher's knowledge that makes him able to facilitate the student's understanding of the material profoundly and sequentially through knowing the student's correct and incorrect concepts and his knowledge of the appropriate methods and strategies (Kleickmann et al., 2013).

Many studies have been done in the MPCK. For example, Hussein (2013) found that the average level of mathematical knowledge mastery among student

teachers was low. El-Mutreb and Al-Salouli (2015) found that many teachers need to have the depth of geometric knowledge they need to teach it well, understand students' mistakes, or judge the validity of their non-traditional ways of solving problems and the possibility of generalization. Ní Shúilleabháin (2016) found that the average level of mathematical knowledge is master.

Khakbaz (2016) showed that three contextual topics affected PCK for teaching mathematics at the university level: The nature of mathematics subjects, university teachers' features, and terms of the learning environment. El-Mutreb et al. (2017) concluded that many teachers needed sufficient knowledge to teach numbers and arithmetic operations and to understand and evaluate students' mistakes. Abdel-Alal (2017) showed a low level of mathematical knowledge needed for teaching. DANÄ et al. (2018) concluded that the PCK of mathematics teachers in public secondary schools about probabilities needs to be revised and that their CK is directly reflected in their teaching processes. Maryono et al. (2017) concluded that experienced teachers with the most detailed pedagogical knowledge can implement teaching practices related to linear equations and change the attitudes and behaviors of students toward dealing with these equations.

Jabr and Al-Zoub (2018) concluded that STEM activities and metacognitive thinking positively impact the development of mathematics teachers' PCK and self-esteem. Hassan et al. (2018) found the effectiveness of the SCAMPER strategy in developing MPCK and students' decision-making skills. Al-Salouli and bin Saud (2018) concluded that the level of mathematical knowledge necessary to teach mathematical subjects was high in all areas: numbers, algebra, geometry, statistics, and probabilities. Budayasa et al. (2018) concluded that there is a weakness in novice teachers' ability to analyze students' causes of difficulties, errors, and misconceptions, that teaching experience is an essential factor in PCK for mathematics teachers, and that Knowledge of the subject matter, Pedagogical Knowledge, and students' Knowledge are essential parts of PCK. Lo (2020) found that many mathematics teachers knew well about students' mathematical misunderstandings, but most needed help identifying these errors. Moh'd et al. (2021) indicated that the level of mathematics teachers' practice of PCK is average, and the study of Spangenberg (2021) indicated that there is difficulty in teaching mathematics due to insufficient PCK. Guler and Celik (2021) showed the effect of the Elective Algebra Teaching Course on Prospective Mathematics Teachers' Pedagogical Content Knowledge. Moh'd et al. (2021) also indicated that the PCK competencies of mathematics teachers need to be better to impact their designed lesson plans (DLP). Jiménez Sierra et al. (2023) explained the importance of developing the PCK of the teacher through the study of the lesson.

Components of PCK

According to Maryono et al. (2017), PCK has three main components: Knowledge of topics regularly taught in one's subject area, knowledge of forms of representation of those ideas, and knowledge of students' understanding of the topics.

Spangenberg (2021) determined PCK in three components: knowledge of the subject matter, knowledge of teaching strategies, knowledge of students' conceptions, and knowledge of the curriculum. In 1990, Grossman developed a structure for PCK that includes four main components: a conception of teaching purposes, which refers to knowledge and beliefs about the aims of teaching-learning topics at different levels; knowledge of students, including Knowledge about Understanding, perceptions, and misconceptions that students have about specific topics related to the subject area; and curricular knowledge, which includes knowledge of instructional strategies and representations for teaching particular topics (Odumosu and Fisayi, 2018).

DANÄ et al. (2018) indicated that secondary school teachers' PCK consists of content Knowledge, curriculum Knowledge, student Knowledge, and Knowledge of teaching methods and strategies, while Grieser and Hendricks (2018) stated that most scholars' models of PCK agree that it includes teacher transformation of knowledge; it consists of four common elements: Knowledge of Students' Understanding, Knowledge of Instructional Strategies and representations, Knowledge of curricula, and the teacher's values and beliefs about Education. Finally,

the technology dimension was integrated to correlate these three components of knowledge (Technology, pedagogy, and content), considering technological knowledge as an entity inseparable from the teacher's PK and content. Thus Technological Pedagogical Content Knowledge (TPCK) is formed (Mishra and Koehler, 2006).

The researchers agreed on the need for complementarity between PK and scientific knowledge of mathematics content (Maryono et al., 2017). Many studies have been conducted on the relationship between MCK and MPCK, such as Campbell et al. (2014), which concluded a positive and direct correlation between teachers' mathematical content and pedagogical knowledge and student achievement in mathematics. Depaepe et al. (2015) revealed a gap in teachers' MCK and MPCK levels and a positive correlation between them. Kleickmann et al. (2015) concluded that CK and PCK represent two distinct dimensions, but they are interrelated. Moreover, teachers demonstrate a high degree of them during service. Lemonidis et al. (2018) showed that teachers' high performance in the knowledge of mental arithmetic content is positively affected by the diversity of using different teaching strategies and the associated pedagogical knowledge.

Depaepe et al. (2018) indicated the effectiveness of the experimental intervention in stimulating MCK and PCK with rational numbers. Kirikçilar and Yildiz (2018) indicated that some mathematics teachers have content and technology knowledge but need to use it effectively, and some teachers need PK and CK. Lee et al. (2018) found that mathematics teachers have SMK related to problem posing and identification, but their results in problem posing do not reflect SMK well, and the teaching practices of mathematics teachers do not reflect their knowledge of content and teaching KCT and KCS well. Odumosu and Fisayi (2018) concluded that there is no significant effect on the interaction between CK and PK. Agathangelou and Charalambous (2021) have shown that mathematical content knowledge (MCK) is a pre-requisite of pedagogical content knowledge (PCK).

RESEARCH METHOD

To achieve the research objectives and answer the research questions, the researcher adopted the descriptive analysis approach; through it, the level of MCK and its relationship to MPCK were measured among secondary school mathematics teachers in the Kingdom of Saudi Arabia.

Research sample

The primary research sample consisted of 49 mathematics teachers at the secondary level in government secondary schools affiliated with the education offices in Dammam and Al-Khobar Governorates, representing 30% of the original population of the study. They were chosen randomly from the research population, which was 166 teachers.

Tools of the research

Mathematical Content Knowledge Test: The test was limited to measuring intermediate and secondary school mathematics teachers' MCK in light of the specialized standards issued by the Education Evaluation Commission. The statement of the test was formulated; it consists of 42 statements of multiple-choice questions containing four alternatives. The teacher should choose the appropriate alternative. These statements are divided into the following: numbers and their operations (6 statements), principles of algebra and real functions (8 statements), Calculus and its applications (5 statements), geometry concepts and theories (13 statements), measurement, its units, and applications (2 statements), statistics concepts and probability and their applications (4 statements), discrete mathematics, logic, and mathematical reasoning (4 statements). A preliminary form of the test was presented to a panel of experts specializing in curricula and methods of teaching mathematics, measurement, and evaluation. Some modifications were made in light of their opinions. The test was applied to a random pilot sample of 16 secondary school mathematics teachers, and the reliability of the test was calculated using the split-half method; it reached (0.970), as well as the subjective validity coefficient of (0.985), which is a high value and indicates that the test has a high degree of reliability. The internal consistency ranged between 0.535 and 0.925, significant at 0.05 and 0.01. The correlation coefficient between the score of each test domain and the total score was also calculated; it ranged between 0.640 and 0.948; they are statistically significant at the significance level (0.05 and 0.01).

The test's ease and difficulty coefficients ranged from 0.25% to 0.75. In addition, the discrimination coefficient was determined for all test items; it ranged between 0.50 and 1.00, which indicates that all items of the test have discriminatory ability.

A Scale of MPCK: The scale was limited to measuring secondary school teachers' MPCK and skills in light of the professional standards issued by the Education Evaluation Commission. After reviewing the literature related to PK, the scale consisted of 24 questions representing a set of situations that describe some of the teaching practices; each question is followed by four different alternatives, and the teacher should choose the appropriate alternative. The scale includes six sub-criteria: Knowledge of the student and learning (4 items), knowledge of specialized teaching methods and general teaching methods (5 items), planning and implementing academic units (5 items), creating interactive and supportive learning environments for students (5 items), and student performance evaluation (5 items). A preliminary form of the scale was presented to a panel of experts specializing in curricula and methods of teaching mathematics, measurement, and evaluation. Some modifications were made in light of their opinions. The scale was applied to a random pilot sample of 16 secondary school mathematics teachers outside the primary study sample, and the responding time of the scale was calculated, which was 45 minutes. The reliability of the test was also calculated using Cronbach's Alpha (α) coefficient, which was 0.955, which is a high degree, and according to the subjective validity coefficient (0.977), which is a high value, indicating that the scale has a high degree of reliability. The internal consistency validity coefficients ranged between 0.578 and 0.915; they are significant at 0.05 and 0.01, and the correlation coefficients were calculated between the criteria of the scale with each other and the total score of the scale; they ranged between 0.642 and 0.951; They are statistically significant at the significance level of 0.05 and 0.01. The grading system for the MPCK scale has been determined according to a quaternary gradation. So that the teacher is given (4) high marks for the first alternative, (3) marks for the second alternative, (2) marks for the third alternative, and a mark for the fourth alternative.

RESULTS AND DISCUSSION

First: Results related to determining the level of MCK of secondary school Mathematics teachers

To answer the first question of the research, "What is the level of MCK among secondary school mathematics teachers?" means and standard deviations for the level of MCK among secondary school mathematics teachers were calculated. The level classification standard was determined as follows: Low level (0-0.499), intermediate level (0.500-0.799), and high level (0.80-1.00), as shown in Table 1.

Table 1: Means and standard deviations for the level of MCK among secondary school Mathematics teachers

Criterion	Mean	Standard Deviation	Level	Rank
Principles of algebra and real functions.	0.674	0.449	Intermediate	1
Measurement, units, and applications.	0.612	0.491	Intermediate	2
Geometry concepts and theories.	0.586	0.455	Intermediate	3
Discrete Mathematics, Logic, and Mathematical Reasoning.	0.556	0.471	Intermediate	4
Numbers and their operations.	0.490	0.480	Low	5
Calculus and its applications.	0.478	0.475	Low	6
Statistics concepts and probability and their applications.	0.404	0.492	Low	7
Overall Mean	0.555	0.468	Intermediate	

Table 1 shows that the means of the MCK test criteria ranged between 0.404 and 0.674, with an overall mean of 0.555, a percentage of 55.5%, and a standard deviation of 0.468, which indicates an intermediate level of MCK among secondary school Mathematics teachers in Dammam and Al-Khobar Governorates. The criterion "Principles of Algebra and Real Functions" had the highest mean (0.674), with a standard deviation of (0.449), then the criterion "Measurement, Its Units, and Applications" with a mean of (0.612) and a standard deviation of (0.491), and the criterion "Calculation and Integral

Calculus and its Applications" with a mean of (0.478) and a standard deviation of (0.475), While the criterion "Statistics Concepts and Probability and their Applications" had the lowest mean (0.404) and a standard deviation of (0.492).

It means that secondary school Mathematics teachers' MCK was intermediate. This result may be due to teachers' lack of motivation, a lack of readiness for such a test because it is not related to professional development, and the unprofessional habits practiced by some teachers, which cause them to lose a lot of knowledge and skills, such as focusing only on teaching a specific class annually and a lack of opportunity to rotate the courses among the teachers of the same school. Also, when the teacher plans the lessons, he confines himself to superficial knowledge and the main ideas presented in the course and needs to expand on them scientifically. He also depends on understanding and application levels in asking questions and avoids probing questions and open answers that need justifications. Teachers' intermediate degree of MCK may also be due to the teacher's failure to practice higher-order thinking skills, the lack of training programs and Arabic references in the field of pure mathematics, the scarcity of websites and resources in mathematics, and the epistemological perceptions of conceptual knowledge (the opinions and ideas that mathematics teachers have about mathematical, conceptual knowledge, its structure, its origin, and how it is formed, acquired, taught, learned, and evaluated) (Al-Khazim, 2020).

The criterion (principles of algebra and real functions) had the highest mean (0.674), and this may be due to the following reasons: When we refer to the matrix of range and sequence of secondary-stage mathematics curricula, we find that the lessons of the principles of algebra and real functions have appeared as a basic structure in mathematics curricula since the beginning of the intermediate stage, based on the concepts of numbers and operations, which had the most significant focus in the elementary school mathematics curriculum. The criterion (numbers and their operations) had a mean of (0.490) at a low level (close to intermediate); Because most of the questions in this theme were based on thinking skills, just as the concepts of algebra and real functions in modern mathematics curricula did not differ much from their predecessors,

The criterion (Statistics concepts and probability and their applications) had the lowest mean (0.404), and this may be since Some concepts appeared with more significant expansion in statistics and probabilities in modern mathematics courses and were not present in previous courses, such as Pascal's triangle, binomial, permutations, and combinations, and the delay in statistics and probability lessons when tracking the matrix of the range and sequence of mathematics curricula to be the first central theme (analyzing and summarising a group of data) in the second intermediate grade (eighth grade). These results are consistent with other studies investigating mathematics teachers' practices in conceptual and procedural knowledge and problem-solving. The balance in conceptual and procedural knowledge teaching practices among intermediate-stage mathematics teachers came at an intermediate performance level (Al-Hlaisi and Al-Salouli, 2016).

These results are also consistent with Al-Shammari et al.'s (2019), which indicated an intermediate performance level for mathematics teachers' practice of conceptual comprehension, while it was weak in each of procedural fluency, strategic competence, adaptive reasoning, productive desire, and a weak level of mathematical Proficiency as a whole, which confirmed that teachers have an intermediate level of conceptual knowledge of calculus and integration topics in the secondary stage. While it differs from the results of other studies that indicated that the level of teaching practices of procedural knowledge among intermediate school mathematics teachers came to a high degree (Al-Hlaisi and Al-Salouli, 2016) and that the level of conceptual Knowledge, procedural Knowledge, and mathematical thinking among mathematics teachers was very low (Qutaish and Al-Fayez, 2019).

Generally, These results confirm the difficulties in the mathematics content areas related to numbers, numerical sense, measurement, patterns, algebra, geometry, and the probability and statistics content areas of mathematics (Ebio, 2022).

Second: Results related to determining the level of MPCK among secondary school Mathematics teachers

To answer the second question of the research, "What is the level of MPKC among secondary school mathematics teachers?", means and standard deviations for the level of MPCK among secondary school mathematics teachers were calculated, and an educationally acceptable standard was used to judge the level of the statements: Low level (1-2.00), intermediate level (2.01-3.00), high level (3.01-4), as shown in table (2).

Table 2: Means and standard deviations for the level of MPCK among secondary school Mathematics teachers

Criterion	Mean	Standard Deviation	Level	Rank
Planning and implementing academic units.	3.326	0.912	High	1
Knowledge of students and learning.	3.102	0.997	High	2
Evaluating students' performance.	2.845	0.948	Intermediate	3
Creating interactive and supportive learning environments for students.	2.825	0.911	Intermediate	4
Knowledge of specialized teaching methods, curriculum, and general teaching methods.	2.796	0.839	Intermediate	5
Overall Mean	2.974	0.918	Intermediate	

Table (2) showed that the means of the criteria for the MPCK scale ranged between 3.326 and 2.796, with an overall mean of 2.974, a percentage of 74.35%, and a standard deviation of 0.918, which indicates an intermediate level in MPCK among secondary school Mathematics teachers in Dammam and Al-Khobar Governorates. The criterion "Planning and implementing academic units" had the highest mean (3.326), with a standard deviation of (0.912), followed by the criterion "Knowledge of the student and learning," with a mean of (3.102), and a standard deviation of (0.997). In contrast, the criterion "Knowledge of specialized teaching methods, curriculum, and general teaching methods" had the lowest mean (2.796) and a standard deviation of (0.839), preceded by the criterion "Creating interactive and supportive learning environments for students" with a mean of (2.825), and a standard deviation of (0.911).

It means that the degree of secondary school mathematics teachers' MPCK was intermediate; this may be due to the recent focus of the Ministry of Education on providing a variety of quality educational programs, such as the Mathematics Curriculum Development Project, which includes five diverse programs: "The Philosophy of Developed Mathematics Curricula," "Active Learning in Teaching Mathematics," "differentiated Teaching in Mathematics," "conceptual comprehension," and "teacher-centered evaluation." In addition, the "planning for understanding," "manual sensibilities in teaching mathematics," "modern and special technical software and aids in teaching mathematics, such as the graphing machine," and "Educational tablet applications in teaching mathematics" programs Othman and Al-Abed (2018) and Hassan et al. (2018) indicated the effectiveness of some training programs, for example, "the SCAMPER strategy of thinking on developing the teacher's knowledge necessary for teaching mathematics," the great interest in professional learning communities and communication within those professional communities, and the variety of supervisory methods that are provided to the mathematics teacher to support and assist him in professional development, such as applied and exchange lessons, workshops, and guided readings. In addition, the teachers' intermediate level of MCK in secondary school According to Lo (2020), a deep understanding of the theory of prime numbers in particular and general mathematics is a precondition for developing the pedagogical knowledge necessary for teaching mathematics.

The criterion (Planning and implementing academic units) had the highest mean (3.326) (high level); This may be due to the focus of educational supervision on unit planning, including setting goals and choosing appropriate teaching strategies, activities, tools, and resources, in addition to their recent interest in the (lesson research) approach, which is based

on sound planning for the lesson. While the criterion (knowledge of specialized teaching methods, curriculum, and general teaching methods) had the lowest mean (2.796) (an intermediate level), This may be attributed to the importance of this criterion because teaching is the heart and foundation of the educational process, including knowledge of the general approaches to teaching, basic Knowledge of methods of teaching mathematics, and Knowledge of the basics of employing educational resources and technologies and their developments (Jiménez Sierra et al., 2023; Moh'd et al., 2022).

These results are consistent with Moh'd et al. (2021), which indicated that teachers have an intermediate mastery of the PCK, and DANÄ et al. (2017), which confirmed that the secondary schools' mathematics teachers' PCK needs to be improved. They differ from Al-Salouli and bin Saud (2018), which indicated that the level of MK necessary to teach mathematics topics covered by international tests TIMSS came out at a high level, Hussein (2013), and Abdel-Alal (2017), which indicated that the mastery mean of the MK necessary to teach mathematics among the study sample was at a low level in general, and El-Mutreb et al. (2017), whose results confirmed that many teachers do not have a sufficient cognitive depth to teach numbers and arithmetic operations, understand and evaluate students' mistakes, or judge the correctness of their alternative ways of solving and the possibility of generalizing them. This deficiency is evident in understanding, realizing, and representing concepts of numbers and their operations, despite the ability of many teachers to do the related arithmetic operations.

Third: Results related to determining the relationship between MCK and MPCK among secondary school Mathematics teachers

To answer the third question of the research questions, "What is the relationship between MCK and PCK among secondary school mathematics teachers?" The Pearson correlation coefficient revealed the relationship between MCK and MPCK among secondary school mathematics teachers, as shown in Table 3.

Table 3: The Pearson correlation between MCK and MPCK

Variables	Teachers number	Pearson Correlation	Significance
MCK	49	0.133	0.361
MPCK	49		

Table 3 showed no statistically significant relationship between MCK and MPCK among secondary school mathematics teachers, as the correlation coefficient value was 0.133, which is not statistically significant at 0.05. This result can be attributed to the relationship between the two variables as one of the components of the teacher's knowledge, as each of them is complementary to the other. Kirikçilar and Yildiz (2018) indicated that the teacher could know specialized content without having the PK associated with it, despite many studies showing a relationship between the two variables, And with the study of Agathangelou and Charalambous (2021), which indicated that Mathematics content knowledge (MCK) is a pre-requisite of pedagogical content knowledge (PCK).

This result is consistent with Lee et al. (2018), who indicated that mathematics teachers have SMK related to problem posing and identification. Still, their results in problem posing need to reflect SMK better. This result differs from Lemonidis et al. (2018), which showed that teachers' high performance in the knowledge of mental arithmetic content is positively affected by the diversity of using different teaching strategies and the associated pedagogical knowledge associated with mental arithmetic of decimal numbers. Campbell et al. (2014) indicated a positive and direct correlation between mathematics content and teachers' PK, and Kleickmann et al. (2015) confirmed that CK and PCK represent two distinct dimensions. However, they are still interrelated, and Depaepe et al. (2015) confirm that there was a positive correlation between the two variables and that it is possible to stimulate mathematics content knowledge (CK) and (MPCK), such as the training intervention that came in their study.

CONCLUSION

This study aimed to determine both the Level of MCK and the Level of MPCK, and to determine the relationship between them among mathematics teachers at the secondary stage. The results showed that the Level of MCK and the Level of MPCK related to the content of mathematics was medium, which indicates that teachers possess superficial knowledge and main ideas related to mathematics, and do not expand it scientifically. In addition, many teachers do not have sufficient depth of knowledge to teach numbers and arithmetic operations, and to understand and evaluate students' mistakes. The study also showed that there is no statistically significant relationship between MCK and MPCK of mathematics teachers at the secondary stage, which indicates the importance of the existence of training programs and development projects that focus on enhancing mathematical knowledge and the educational aspect related to teaching it. These results draw attention to the importance of the teacher's possession of the knowledge related to the specialized content and the educational knowledge related to it.

Recommendations and suggestions for further research

In light of the research results, it is necessary to pay attention to educational professional development based on professional standards, whether they are specialized standards or educational standards related to specialization. There is also a need to use a variety of professional development activities, such as attending scientific conferences, educational meetings, and training programs and activating professional learning communities among teachers of the same specialization or with other disciplines through various social media. Building training programs in PCK components and, in addition, reconsidering mathematics teacher preparation programs by focusing on the student teacher's applied practices and theoretical knowledge. This research suggests investigating the impact of a training program on developing MCK and MPCK and investigating the relationship between the level of knowledge of the mathematics teacher and his teaching skills. And the study of the relationship between the teacher's mathematical knowledge level and his attitude toward teaching mathematics.

Limitations of the study

There are many components for each of the PCK and MCK that were referred to by many studies. Still, the study was limited to the components mentioned in the research tools, as well as the small study, population, and size of the sample, as it was limited only to the cities of Dammam and Al-Khobar, and therefore the results cannot be generalized to other cities in the Kingdom of Saudi Arabia. Also, when selecting the sample, the study did not consider the following factors: the teachers' previous background, self-efficacy, and other factors that interact with the PCK components.

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