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RESEARCH ARTICLE

Problems Posed by Preservice Primary Teachers Based on the Four Operations with Fractions and Their Skill of Modelling These Problems

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ABSTRACT

In the current study, it is aimed to examine the problems written by preservice primary teachers based on the four operations with fractions in different problem posing situations (free, semi-structured and structured) and their modelling skills; that is, skills required to convert these problems into visual representations, and to explore their views on the process. The study employed case study design, one of the qualitative research methods. The study group consists of 82 primary teacher candidates selected by using the purposive sampling methods from among the students attending the Departments of Primary Teaching of the Education Faculties of three state universities located in the Central Anatolian region. The participation in the study is on a volunteer basis. Content analysis was conducted on the documents obtained from the question form containing different problem posing situations administered to 82 participants and the data obtained from 32 participants who expressed their views on the process. According to the findings, all the participants were able to pose problems based on the four operations in different problem posing situations (free, semi-structured and structured), and half of them were able to model the problems they posed. The participants most frequently used the area/region model in the visual representations of the problem situations. The problems posed in the study and the preservice primary teachers' views on the problem-posing process are presented in detail.

INTRODUCTION

Primary school, the first step of compulsory education, constitutes the most important part of one's education life. Children spend a long time at school, which places important responsibilities on teachers. Primary teachers play an extremely valuable role in primary schools by addressing children's developmental needs, getting to know them in every way and making the greatest contribution to their cognitive and affective development (Özen and Çakır, 2021). Therefore, it is essential for them to be well-educated in every discipline within the framework of primary education (Doğan and Tertemiz, 2018). One of the disciplines in which preservice primary teachers are expected to be well-equipped is mathematics (Van De Walle et al., 2016). Mathematics is one of the disciplines in which primary teachers must be qualified enough, both because it is a central assessment area in national and international exams and because it has an important place in daily life (Ergül and Doğan, 2018). In addition, primary teachers have key responsibilities such as relating learned knowledge to everyday life, creating a positive classroom environment and using effective teaching methods. Therefore, competence in teaching mathematics is a crucial factor in making students' learning meaningful (Doğan and Tertemiz, 2018).

One of the requirements of being competent in teaching mathematics is to have the ability to develop word problem situations that help students relate the mathematics they have learned to real-life situations and to solve them through models (Kılıç, 2013; McAllister and Beaver, 2012; National Mathematics Advisory Panel, 2008). At this point, it is expected that preservice primary teachers have developed their skills both to pose problems (Barlow and Cates, 2006; Parhizgar et al., 2021) and to express them using models (Akkan et al., 2018). Problem posing and modelling have been investigated by many researchers in the mathematics education community (Akben, 2020; Akkan et al., 2018; Mallart, 2018; Maaß, 2006; Schwarz and Kaiser, 2007; Silver, 1994; Stoyanova and Ellerton, 1996). It is thought that using problem-posing activities in mathematics education helps teachers gain deeper insights into students' understanding and knowledge, and can reduce students' dependence on teachers and textbooks to some extent (Lavy and Shriki, 2007). It is known that the use of mathematical modelling is necessary for individuals to easily understand and solve problems they encounter in daily life (Wood, 1992). However, in order to equip individuals with these skills, it is essential that teachers should have the mastery of knowledge and skills required to pose problems, solve problems and use models (Thomas and Hart, 2010).

It is stated that the number of studies on the problem-posing skills of preservice primary teachers is quite limited (Lua, 2009). Interviews with teacher educators have revealed that they experience significant difficulties in writing meaningful word problems for the given operations (McAllister and Beaver, 2012). It can be said that certain problems persist from the past to the present in word problem-posing activities focusing solely on one of the four operations, especially within the subject of fractions (Akbaba-Dağ and Kılıç-Şahin, 2019; Akçay and Ardıç, 2020; Ball, 1990a, 1990b; Chapman, 2012; Li et al., 2020; Parhizgar et al., 2021; Simon, 1993; Xie and Masingila, 2017). Despite the interest shown by the mathematics education community in incorporating mathematical problem-posing activities into classroom practices, previous research has revealed that not enough emphasis is put on the meanings carried by the problems posed and that the problems are not represented with models. The processes of problem posing and modelling play a critical role in the development of individuals' mathematical thinking and problem-solving skills (Polya, 2002; Kilpatrick, 1987). Problem posing allows students to analyze real-life situations and transform them into a mathematical context (Xie and Masingila, 2017), while modelling involves structuring and expressing this context to explore possible solutions (Schwarz and Kaiser, 2007). The strong relationship between these two processes makes it possible for students to view mathematics not only as a series of operations or rules but also as a tool for creative and analytical thinking (Schukajlow et al., 2012). In this context, addressing problem posing and modelling skills together could lay the groundwork for preservice primary teachers to develop more effective strategies in both abstract and concrete problems. This can help them gain new and deeper insight. In light of all these findings, the current study focuses on the investigation the problem-posing skills of preservice primary teachers regarding the four operations with fractions and also on how the preservice primary teachers model the problems they pose.

LITERATURE REVIEW

Problem Posing

Problem posing is an important component of the mathematics curriculum and is considered the heart of mathematical activities (Brown and Walter, 1993). It involves generating new problems based on an existing one or re-formulating a given problem (Ticha´ and Hosˇpesova´, 2009). Research on problem posing draws different types of frameworks (Coşkun-Doğan, 2019). According to Stoyanova and Ellerton (1996), the problem-posing process is divided into three situations: free, semi-structured and structured. In the free problem-posing situation, students are asked to pose a problem for a real-life situation. For example, asking students to create a problem for their peers to solve, either specifying or not a topic. In the semi-structured problem-posing situation, students are provided with a figure, diagram, picture, table, chart, etc., and asked to create a problem based on it. That is, students are asked to analyze the structure of a semi-structured situation and complete it by using their knowledge, skills, concepts and connections to prior mathematical experiences. In the structured problem-posing situation, students create a problem that aligns with the given symbolic equations or reformulate a given problem statement in their own way (Stoyanova and Ellerton, 1996). Although different frameworks for problem posing have been proposed (Christou et al., 2005; Contreras, 2007; Silver, 1994), the framework developed by Stoyanova and Ellerton (1996) is used in the current study. The three-situation problem-posing framework was deemed sufficient for both understanding the problem-posing skills of prospective elementary school teachers and examining their problem-posing skills related to fractions and the meanings of the four operations.

Problem posing and Mathematics Learning

Problem posing is an important mathematical activity due to its numerous benefits for researchers, teachers, teacher candidates and students (Kesan et al., 2010). Lavy and Bershadsky (2002) emphasized that problem posing is more important than problem solving, which is only one aspect of mathematical or experimental skills. They stated that new questions bring new possibilities, which in turn enhance creative imagination. In addition, the importance of students developing the skill to pose their own problems is emphasized in the publications of the National Council of Teachers of Mathematics (2000). Previous studies in the field of mathematics education point out the importance of problem posing to enhance and assess students' understanding of key mathematical concepts and engage them with meaningful mathematics (Chapman, 2012; Luo, 2009).

According to Stoyanova (2003), problem posing can be used both as a teaching method and a learning activity within mathematics education. When teachers pose problems for students to solve, it becomes a teaching method, while when students pose problems based on their own interests, it becomes a learning activity (Crespo, 2003). In this sense, teachers can use problem-posing tasks to reveal students' mathematical errors or misconceptions, and by using activities such as problem posing with fractions, they can effectively teach mathematical concepts (Barlow and Drake, 2008). It is stated that for preservice primary teachers, problem posing contributes to the development of their pedagogical content knowledge in mathematics education and enhances their understanding of what mathematics education means (Toluk-Uçar, 2009). Moreover, primary teacher candidates who actively and reflectively engage in the problem-posing process can generate meaningful, productive and well-constructed mathematical problems (Contreras, 2007). For students, problem posing fosters the development of mathematical thinking (Silver and Cai, 1996), creativity (English, 2019), verbal skills and contributes to interdisciplinary learning (Kavuncu and Yenilmez, 2021). It also helps in developing a positive attitude towards mathematics (Chapman, 2012), strengthening critical thinking and reasoning skills, enabling the understanding of mathematical concepts (Gür and Aykurtlu, 2021), and provides an environment where misconceptions and errors can be discovered (Tichá and Hošpesová, 2009). Problem-posing activities develop problem-solving and metacognitive skills (Akben, 2020; Chen et al., 2015). It is also suggested that the ability to successfully develop word problems is an indicator of a rich network of conceptual knowledge (Chapman, 2012). Therefore, in the current study, it is expected that the problems posed by preservice primary teachers will also reveal their conceptual understanding of fractions.

Concept of Fractions

The concept of fractions is as old as mathematical thinking, yet operations with fractions are relatively new in the history of mathematics (Gökkurt-Özdemir, 2018). The use of fractions dates back to around 3000 B.C. (among the Egyptians and Babylonians) and extends to the Romans. It is known that the earliest representations of fractions in history trace their origins to the Indians and

Arabs (Olkun and Toluk-Uçar, 2014). A fraction is a numerical representation of one or more equal parts of a whole (Gökkurt-Özdemir, 2018).

The concept of fractions, as a conceptually rich topic in the primary mathematics curriculum with wide applications in everyday life, is also found within other areas of mathematics such as decimal numbers, percentages, ratio, proportion (Alacacı, 2009), probability, algebra and measurement (Van De Walle et al., 2016). Since the concept of fractions, like problem posing, is considered abstract in nature, it is regarded as one of the more difficult concepts to understand in mathematics (Booker, 2013; Dorgan, 1994; Van De Walle et al., 2016). Research has shown that students struggle with many aspects of the subject of fractions (Alacacı, 2009). Some of these challenges include posing problems involving the four operations with fractions (Işık and Kar, 2012), posing and solving problems with fraction models (Kavuncu and Yenilmez, 2021), ordering fractions, errors in operations with fractions (Biber et al., 2013), and converting decimals to fractions (Yıldız, 2017). In fact, the concept of fractions is not well understood even by adults (Van De Walle et al., 2016). Therefore, it is important to uncover the conceptual understanding levels of primary teacher candidates regarding the concept of fractions through problem posing, as they will be the future educators.

Modelling Problems

Problem posing in mathematics instruction plays a central role in developing both students' conceptual understanding and their application skills (Polya, 2002). However, the effectiveness of the problem-posing process depends on the effective use of the skill of modelling problems (Van De Walle et al., 2016). Modelling is a process that allows the connection of mathematical thinking to real-world problems and helps students understand abstract mathematical concepts (Ärlebäck, 2009). This process requires not only correctly applying mathematical operations but also situating these operations within a meaningful context (Lesh and Fennewald, 2010).

Modelling problems provides students with the opportunity to analyze the structure of a problem, identify its key elements, and express these elements through a mathematical model (Schwarz and Kaiser, 2007). This not only helps students understand mathematical concepts but also enhances their ability to apply these concepts to different situations (Albarracin et al., 2022). For example, when working on a fraction problem, students' visualizing the problem, modelling it and transforming it into a mathematical solution process deepens their learning.

In an educational context, the process of modelling problems also offers significant pedagogical advantages. Modelling allows students to experience mathematical thinking in a more concrete way (Doerr and Lesh, 2011). At the same time, it encourages students to engage more actively in the problem-posing process and guides them to think critically about problem situations (Schwarz and Kaiser, 2007). This process not only helps students develop critical thinking and analytical skills, but also provides teachers with a powerful tool to assess and guide students' learning processes (Blum, 2011; Stillman et al., 2013). In summary, modelling problems should be viewed not only as a learning objective in mathematics education but also as a learning process that deepens mathematical understanding and application.

Current Study

Fractions are one of the key subjects in the primary school mathematics curriculum and play a critical role in the development of students' mathematical thinking skills. However, the literature indicates that students often have difficulties in the four operations and problem-solving processes with fractions (Ada et al., 2020; Kavuncu and Yenilmez, 2021; Işık-Tertemiz and Sulak, 2013; Tertemiz, 2017). This highlights the importance of primary teachers' pedagogical competence in teaching abstract mathematical concepts such as fractions. Problem-posing and modelling skills in particular enable teachers to teach these concepts by relating them to concrete contexts for students. In the previous research, there are mostly investigations based on written expressions in subjects such as posing problems with fractions and posing problems according to fraction types and meanings (Akbaba-Dağ and Kılıç-Şahin, 2019; Akçay and Ardıç, 2020; Canbazoğlu and Tarım, 2021; Chapman, 2012; Doğan-Coşkun, 2019; Kılıç, 2013; 2015; Leung and Carbone, 2013; Li et al., 2020; Lu, 2009; McAllister and Beaver, 2012; Rivzi, 2004; Ünlü and Ertekin, 2012; Xie and Masingila, 2017). However, it is suggested that such activities should be supported with interviews (Toluk-Uçar, 2009) and the number of applications to be conducted should be increased (Akbaba-Dağ and Kılıç-Şahin, 2019; Akçay and Ardıç, 2020; Kılıç, 2013; Rivzi, 2004). The current study aims to fill a significant gap in the literature by examining preservice primary teachers' problem-posing and modelling skills based on the four operations with fractions and developing an in-depth understanding of the process through interviews. The findings to be obtained may shed light on the processes of developing their mathematical pedagogical content knowledge and provide recommendations for improving teaching methods in this area. Furthermore, the study can contribute to making mathematics education in primary teacher education programs more effective and thus help students to develop a deeper understanding of fractions. In this context, the study has the potential to contribute not only to theory but also directly to the teaching practice. In these aspects, the study significantly differentiates itself from other research. In this regard, answers to the following research questions will be sought:

1. How are the problems posed by preservice primary teachers on fractions distributed and modelled in terms of the meanings of the four operations?

2. What are the views of preservice primary teachers on problem-posing and modelling regarding fractions?

METHOD

Research Model

The study employed the case study design, one of the qualitative research methods. The basic idea in a case study is to select a situation and explain how the situation illustrates a problem or an issue (Creswell, 2019). The situation can be a single person, a program, a group, an institution, or a society (Merriam, 2018). In the current study, the problem-posing and modelling skills of primary teacher candidates regarding the four operations with fractions will be examined.

Participants

The participant consists of 82 preservice primary teachers who are third-year students in the primary education undergraduate program at three state universities. In the selection of the participants, the criterion sampling technique, one of the purposive sampling techniques, was used. The fundamental logic of criterion sampling is to examine and analyse all cases that meet a set of predetermined criteria (Patton, 2018). In the current study, the criterion used in the selection of the participants was their having the Mathematics Teaching II course because instruction given within the scope of the Mathematics Teaching II course includes the concept of fractions and their teaching, operations with fractions, visual representations of fractions through models, and how the concept of fractions is addressed in the national curriculum. And, for the participants to include in the study, it was thought to be necessary for them to have knowledge of the specified subjects. According to this criterion, a total of 82 participants from three state universities participated in the study on a volunteer basis. Of the participating teachers, 62 are female and 20 are male.

Data Collection Tools

In case studies, various data collection methods can be used, ranging from questionnaires to face-to-face interviews (Merriam, 2018). Accordingly, the data for the study were collected through a problem-posing and modelling form developed by the researchers and a semi-structured interview form. The problem-posing and modelling form includes situations based on the types of problem posing related to the four operations with fractions, as proposed by Stoyanova and Ellerton (1996).

In addition, during the problem-posing and modelling activities, questions from the semi-structured interview form (Appendix 1) were asked to the students, and their views on the problem-posing and modelling process were collected. This allowed for an in-depth understanding and examination of the research topic.

Data Collection

The data for the study were collected simultaneously at the three state universities. The first and second researchers, after teaching the preservice primary teachers the subjects of problem solving and posing and fractions within the scope of the Mathematics Teaching II course at their respective universities, and providing training on the place of these subjects in the primary school mathematics curriculum (Ministry of National Education, 2018) and some teaching practices, distributed the prepared questionnaire. The participants were asked to complete the questionnaire face-to-face within a 50-minute time span. Subsequently, interviews were conducted via email with 32 volunteer participants to gather their opinions about the process. The data from the participants at the third university were collected face-to-face concurrently with the data collection at the other two universities through a colleague working in this university's undergraduate primary education program (by the faculty member teaching the Mathematics Teaching II course in this department), and interviews with the participants were collected via email by the first researcher. The process of collecting all the data took approximately three months.

Data Analysis

The data of the study were analyzed using the content analysis method. Firstly, the problem-posing and modelling forms were coded as "PPT 1, PPT 2, PPT 3 … PPT 82" (PPT= Preservice Primary Teacher). Then, the posed problems were analyzed based on the meanings carried by the four operations - "joining, separating, comparing, equal groups and multiplicative comparison" - as described by Van De Walle et al. (2016) and Olkun (2023). For unanswered cases that could not be classified into any group, the researchers identified the theme of "inability to pose a problem." The findings of this section were expressed as frequencies (f), and percentage values (%) were calculated. Subsequently, models capable of transforming word problems into visual representations were examined to determine whether the participants were able to create models that corresponded to the problems they wrote and their distribution across the models used in the solutions (area/region, number line, volume, set) was analyzed. The findings of this section are expressed as frequency (f) values and percentage values are calculated due to the variability of visual representation types used by individuals in their problem-posing processes.

In the content analysis conducted on the opinions expressed by the preservice primary teachers on the problem-posing and modelling processes, data from 32 volunteer participants were analyzed. The analysis stages defined by Creswell (2019) were followed during the content analysis process of these data. Accordingly, the data obtained from the interviews were first subjected to a preliminary analysis for verification. Subsequently, each piece of data was reviewed through preliminary reading, and brief notes were taken. At this stage, it was also checked whether there was any data loss. For this purpose, it was first checked whether complete data had been obtained from all the 32 participants. Then, the obtained data were examined to verify whether they contained responses relevant to the questions asked. The checks revealed that there were no missing data. Subsequently, the data were subjected to a second and more detailed reading. During the second reading, codes were determined using the brief notes taken during the first reading. After the researchers independently completed the processes of coding and theme generation, the generated themes were compared for similarities and differences. For some cases where differences occurred, themes that seemed more logical and consistent with the content were accepted. In some cases, an agreement could not be reached, and assistance was sought from another academic expert in qualitative data analysis during the data analysis stage. As a result of the third academic's review, codes and themes that were considered more appropriate were accepted, frequency distributions were calculated, and thus the analysis processes were completed. For each case, direct quotations from teacher candidates' statements are given. The names of the themes derived from the data analysis of the interviews and sample quotations are presented in a table format (Table 1).

Table 1: Themes and sub-themes derived from the interviews and sample quotations

Themes	Sub-themes	Quotations
Benefits of Problem Posing	Student Development	"I think that problem-posing activities in math lessons are beneficial. As I pose more problems, my problem-solving speed increases."
	Creativity and Critical Thinking	"It was an activity that supported critical thinking skills through elaborate thinking and opened up the person's mind."
	Relevance to Real Life	"I think that problem-posing activities in math lessons are beneficial because these activities help us solve many problems that can also be used in daily life."
Benefits of Problem Posing with Fractions	Conceptual Learning	"The problem-posing activity related to fractions helped me better understand the logic of fractions."
	Recognizing Deficiencies	"Through fraction problems, I realized that there is an area where I need to address my deficiencies..."
	Development of Teaching Skills	"Working with fraction problems was very valuable for the improvement of my instructional methods. I saw how I can make the topic more concrete."
Problem Posing Type Preferences	Free Problem Posing	"I feel more creative in free problem posing because our imagination is not restricted."
	Structured / Semi-structured Problem Posing	"I felt more competent in structured and semi-structured types of problem posing because the specific rules and points to pay attention to guided me..."
Competences in problem posing with fractions	The Need for Self-Development	"I have difficulties in understanding problems at a complex level; I need to improve myself in this area..."
	Medium Competency Level	"My problem-posing skill is not very developed, but I think it is at a medium level."
	Competence in Free Problem Posing	"I felt more comfortable in free problem posing because I didn't push myself too hard."
Model Choice and Alternatives	Easiness of Modelling	"I preferred these models because their drawings were easy to produce."
	Suitability of the Problem	"I used the area model because it visually made it easier to understand the topic."
	Alternative Models	"If I had used a tablet, it would have been easier for me to draw, so I might have preferred different models."

As seen in Table 1, the data obtained from the interviews are grouped under 5 themes and 14 sub-themes. The frequency values for the themes and sub-themes are provided. In addition, examples from the statements of teacher candidates are provided as the basis for the themes.

Validity and Reliability

A series of procedures were carried out to ensure the validity and reliability of the study. Prior to the development of the interview and problem-posing/modelling forms, the questions were subjected to the review of two experts in the field (Başkale, 2016). Secondly, the data analyses were conducted by both researchers, adhering to the data analysis template, and mutual checks were performed (Denzin and Lincoln, 2011). Thirdly, this categorization was presented to the review of another academician (Lincoln and Guba, 1985). As a result of these checks, the final categories were created. Fourthly, a direct quotation or example case appropriate for each category was provided in the findings section (Arastaman et al., 2018). The agreement between the researchers was calculated using Cohen's Kappa coefficient. Cohen's Kappa coefficient is a statistical method used to determine the actual agreement between two evaluators by controlling for random agreement between observations (McHugh, 2012). The Cohen's Kappa coefficient of 0.95 obtained in this study indicates that the evaluations of the two researchers are highly consistent.

FINDINGS

The Distribution and Modeling of The Problems Posed on The Subject of Fractions in The Free Problem-Posing Category in Terms of the Meanings of the Four Operations

The first finding of the study is related to how the problems posed by the preservice primary teachers in the free problem-posing category regarding fractions are distributed in terms of the meanings of the four operations. They were asked to pose a problem by presenting the problem-posing situation, "Pose a problem that will be challenging for your friend and model it." The obtained findings are presented in Figure 1.

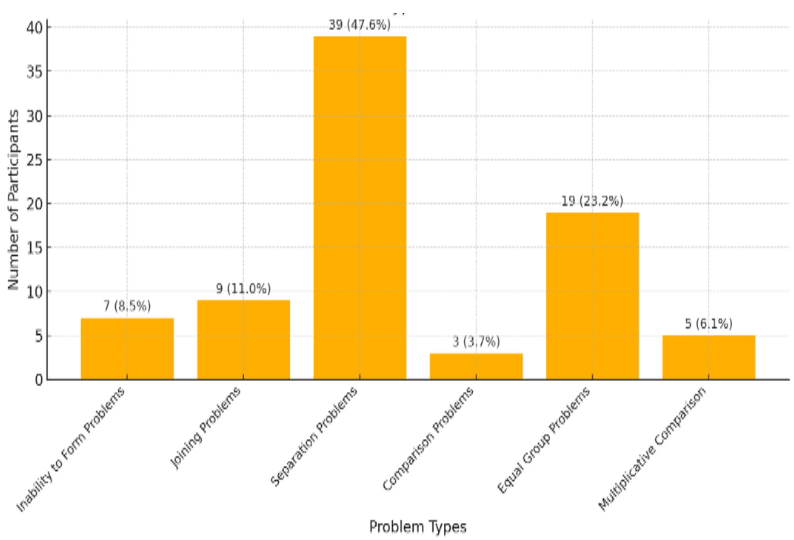


Figure 1: Combined analysis of student responses distribution for free problem posing

According to Figure 1, the participants most frequently posed problems related to the separation meaning of subtraction in the free problem posing situation, which was the first problem posing situation given to them (f=39, 47.6%), followed by the problems related to the equal groups meaning of the division operation (f=19, 23.2%), the problems related to the joining meaning of the addition operation (f=9, 11%), the problems with the multiplicative comparison meaning of the multiplication/division operations (f=5, 6.1%) and the problems including the meaning of comparison (more or less) and that can be solved with the addition and subtraction operations (f=3, 3.7%). The number and percentage of participants who were unable to pose a problem suitable for the free problem-posing situation were low (f=7, 8.5%).

The preservice primary teachers demonstrated their ability to transform the word problems they created in the context of open-ended problem-posing into visual representations using models. More than two-thirds of participants successfully modeled the problems they had constructed (f = 58, 70.7%). However, it was observed that less than one-third were unable to model the problem situations they had posed (f = 24, 29.3%). Among the problems generated in the open-ended problem-posing category, the area model was the most frequently utilized for visualization (f = 51, 87.9%). The set model was employed by a small number of participants (f = 6, 10.3%). Meanwhile, the number line/length model was preferred by only one participant (f=1, 1.7%) as a visual representation of their self-generated problems.

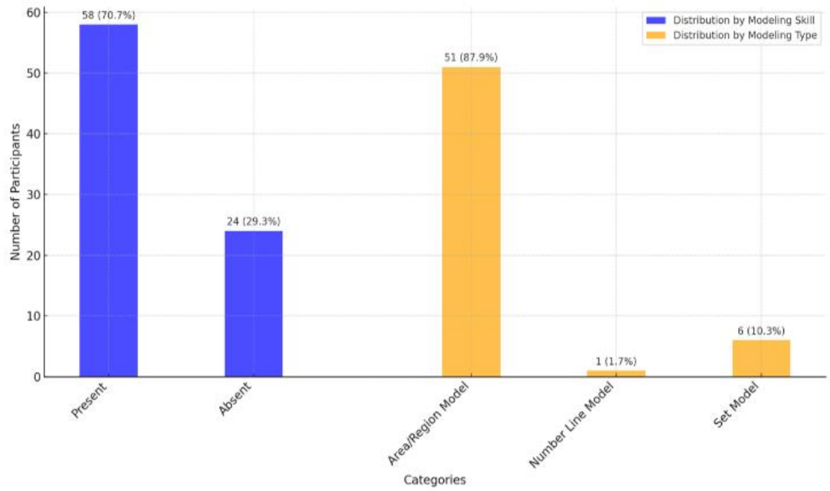
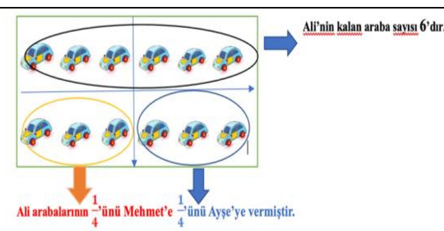
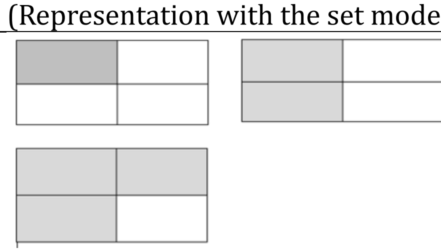
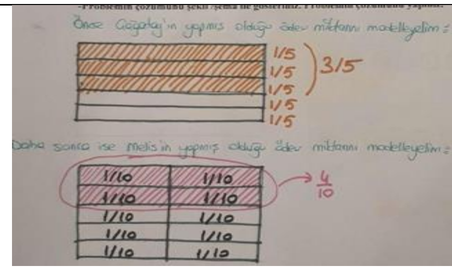
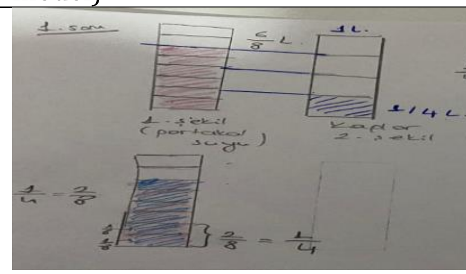
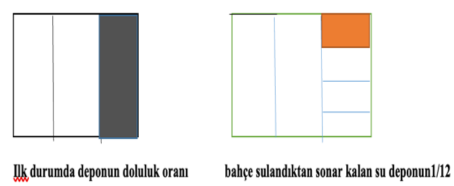


Figure 2: The modelling status and model type distributions of problems posed in the free problem-posing situation

Examples of the distributions in the free problem-posing (Figure1) and modelling (Figure 2) forms expressing the meanings of the four operations by the preservice primary teachers are presented in Table 2.

Table 2: Examples of free problem posing and modelling

The Meanings of the Four Operations	Examples of Free Problem Posing Situation	Examples of Modelling
Separation Problems	"Ali distributed his 12 toy cars to his friends. If Ali gave 1/4 of his cars to Mehmet and 1/4 to Ayşe, how many cars does Ali have left?" (PPT 7)	 (Representation with the set model)
Joining Problems	"Ali eats 1/4 of a whole cake first and then 1/2 of the whole cake. How much cake has Ali eaten in total?" (PPT 12)	 (Representation with the area/region model)

Comparison Problems	"Çağatay and Melis are working on an assignment. Çağatay has completed 3/5 of the assignment, while Melis has completed 4/10 of it. How much more of the assignment does Melis need to complete to have done the same amount as Çağatay?" (PPT 33)	 (Representation with the area/region model)
Equal Groups Problems	"6/8 litres of orange juice are to be poured into containers of 1/4 litre each. How many of these containers will be filled?" (PPT 28)	 (Representation with the area/region model)
Multiplicative Comparison Problems	"Mrs. Ayşe watered her garden with 3/4 of the remaining water in a tank that was 1/3 full. Then, 7/11 of the empty space in the tank was filled with water. What is the ratio of the initial amount of water in the tank to the final amount of water?" (PPT 40)	 (Representation with the area/region model)
Inability to Pose Problems	Ability: Absent (PPT 1)	Ability: Absent

The Distribution and Modeling of The Problems Posed on The Subject of Fractions in The Semi-Structured Problem-Posing Category in Terms of the Meanings of the Four Operations

Figure 3 represents the results of problem posing and modelling in the semi-structured (including the adjustment sub-dimension) problem-posing situation for the four operations with fractions. The preservice primary teachers were asked to pose problems suitable for the semi-structured problem posing situation given below.

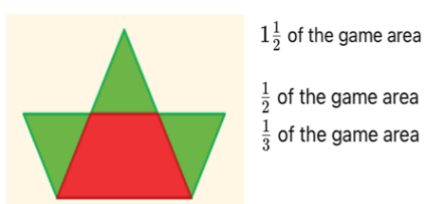


Figure 1. Game Area

The fractional expressions represented by Figure 1 above illustrate a game area. Design a problem related to the game area. Provide the problem by demonstrating it using the model.

As seen in Figure 3, nearly one-third of participants were unable to successfully pose a problem in the semi-structured problem posing situation (f=23, 28%). When the problems posed in this category are examined, it is seen that the problems posed in the semi-structured situation mostly have the separation meaning of the subtraction operation (f=18, 22%), followed by the problems posed for the equal groups meaning of the division operation (f=14, 17.1%), the problems posed for the joining meaning of the addition operation (f=12, 14.6%), the problems including the meaning of comparison (more or less) and that can be solved with the addition and subtraction operations (f=8, 9.8%) and the problems with the multiplicative comparison meaning of the multiplication/division operations (f= 7, 8.5%).

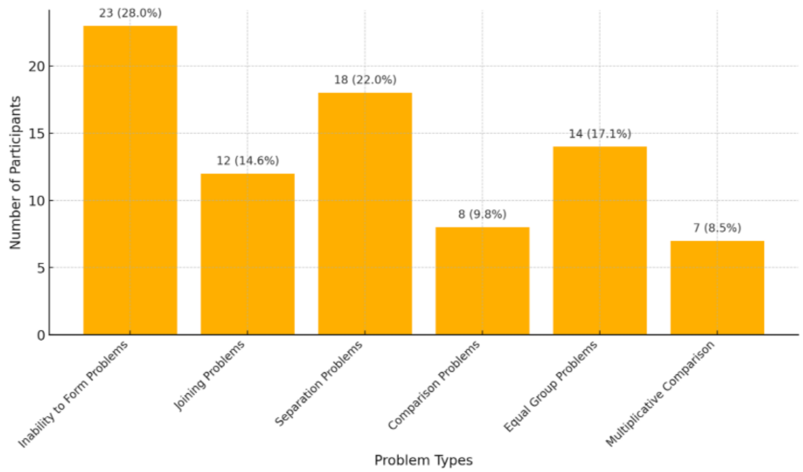


Figure 3: Combined analysis of student responses distribution for semi-structured problem posing

Figure 4 shows the success of the preservice primary teachers in modelling the problem situations given in the semi-structured situation. Half of participants were able to create correct models for the problems they posed in the semi-structured problem posing situation (f=46, 56.1%). It was seen that nearly the other half of participants were unable to model the problem situations they posed (f=36, 43.9%). The area/region model is the most commonly used model among the modelling types (f=42, 91.3%). It is seen that a small number of participants used the set model to model the problems they posed in the semi-structured type (f=4, 8.7%).

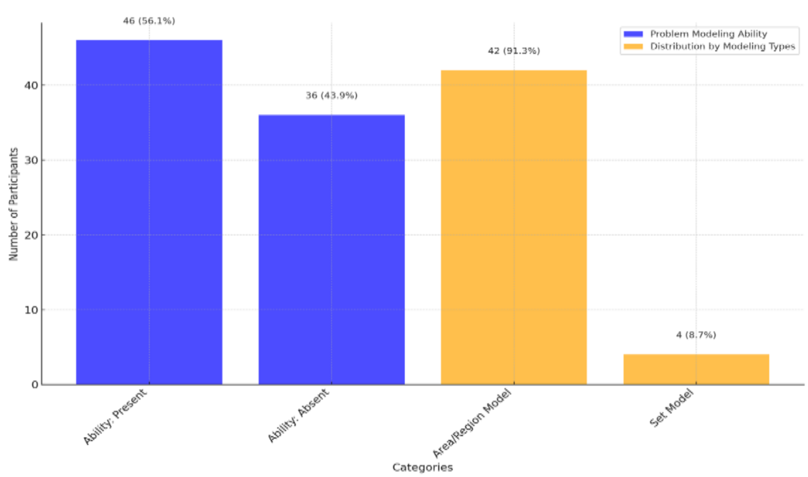
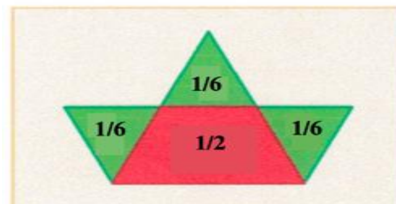
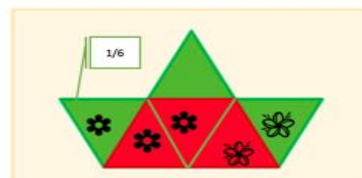
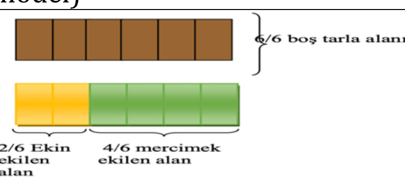
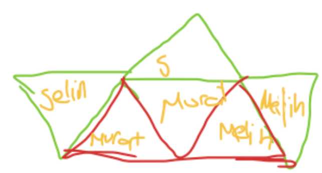
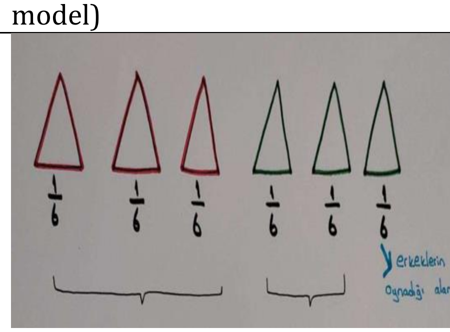


Figure 4: The modelling status and model type distributions of problems posed in the semi-structured problem-posing situation

Examples expressing the problems posed and their modelling (Figure 4) in the semi-structured problem-posing situation (Figure 3) by the preservice primary teachers are presented in Table 3.

Table 3: Examples of semi-structured problem posing and modelling

The Meanings of the Four Operations	Examples of Semi-Structured Problem Posing Situation	Examples of Modelling
Separation Problems	"Dilara has seen that a new play area has been created next to her house. On the map of the play area, some part of the area is coloured red, and some part is coloured green. Each green area covers 1/6 of the park. Since the game is played only in the red area of the park, what is the size of the area where no games are played?" (PPT 36)	 (Representation with the area/region model)
Joining Problems	"In the play area shown above, roses will be planted on 1/2 of the area, and daisies will be planted on 1/3 of the area. What is the total area where flowers are planted?" (PPT 39)	 (Representation with the area/region model)
Comparison Problems	"Kadir has planted crops in 1/3 of his field, which consists of 6 equal parts. Since he planted lentils in the remaining area, how much less is the area planted with lentils than the entire field?" (PPT 44)	 (Representation with the area/region model)
Equal Groups Problems	"There is a play area divided into 6 equal parts with coloured sections. Selin will play in the green sections and Murat will play in the red sections. When Melih joins them, the three of them want to share the play area equally. How much area will be allocated to Melih?" (PPT 12)	 (Representation with the area/region model)
Multiplicative Comparison Problems	"Teacher Umay wants her students to play a game in the classroom and has shown the available spaces in the diagram above. Since there are more female students, she allocated 1/2 of the play area to them. She also allocated 1/3 of the play area for the game materials. What fraction of the play area is allocated to the male students? " (PPT 35)	 (Representation with the set model)

Inability to Pose Problems	Ability: Absent (PPT 82)	Ability: Absent

The Distribution and Modeling of The Problems Posed on The Subject of Fractions in The Structured Problem-Posing Category in Terms of the Meanings of the Four Operations

Figure 5 represents the results of the structured problem posing (including the selection sub-dimension) and modelling related to the four operations with fractions. The preservice primary teachers were presented with structured problem-posing situations requiring addition, subtraction, multiplication and division and were asked to pose a problem suitable for one of them.

3. Choose one of the incomplete story scenarios below and write a problem. Demonstrate the problem using a model.

a) Donations are being collected under two categories for a relief campaign. Food donations constitute 2/9 of the total contributions. …………… constitutes the remaining portion as clothing donations. Complete the problem scenario so that the answer is 1/9.
b) A bakery has 2 whole loaves and 1/2 loaves of white bread dough and ..... loaves of whole wheat bread dough. Complete the problem scenario so that the answer is 37/6.
c) On a table, there is a mixture of 5 whole and 1/6 bottles, along with bottles of ..... volume. Complete the problem so that the answer is 32.
d) On the counter, there is 3/4 of a tres leches cake. This cake's .... Complete the problem so that the answer is 3/8.

As seen in Figure 5, when the problems posed by the participants are ranked from most to least common, it is observed that the problems with the equal groups meaning of the division operation are the most common (f=20, 24.4%), followed by the problems with the separation meaning of the subtraction operation (f=17, 20.7%), the problems with the joining meaning of the addition operation (f=16, 19.5%), the problems with the meaning of comparison (more or less) and that can be solved with the addition or subtraction operations (f=11, 13.4%) and the problems with the multiplicative comparison meaning of the multiplication and division operations (f=4, 4.9%). Finally, it was seen that a small number of participants were unable to pose problems suitable for the structured problem posing situation (f=14, 17.1%).

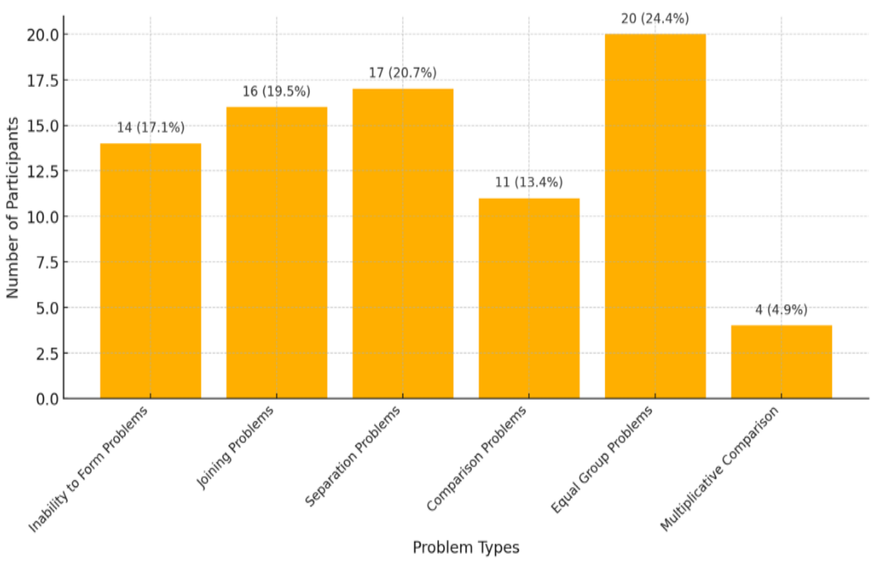


Figure 5: Combined analysis of student responses distribution for structured problem posing

The Figure 6 illustrates the modeling status of problems constructed by the preservice primary teachers in alignment with structured problem situations designed for the four operations with fractions, as well as the distribution of visual representation types utilized during the modeling process. According to the findings, the participants were able to model the problems they created for structured problem situations to a significant extent (f = 48, 66.7%). However, it was observed that slightly more than one-third of participants were unable to use a model for their problems (f = 24, 33.3%). The distribution of the types of models used for visually representing the problems indicates that the area/region model was overwhelmingly preferred (f = 48, 100%).

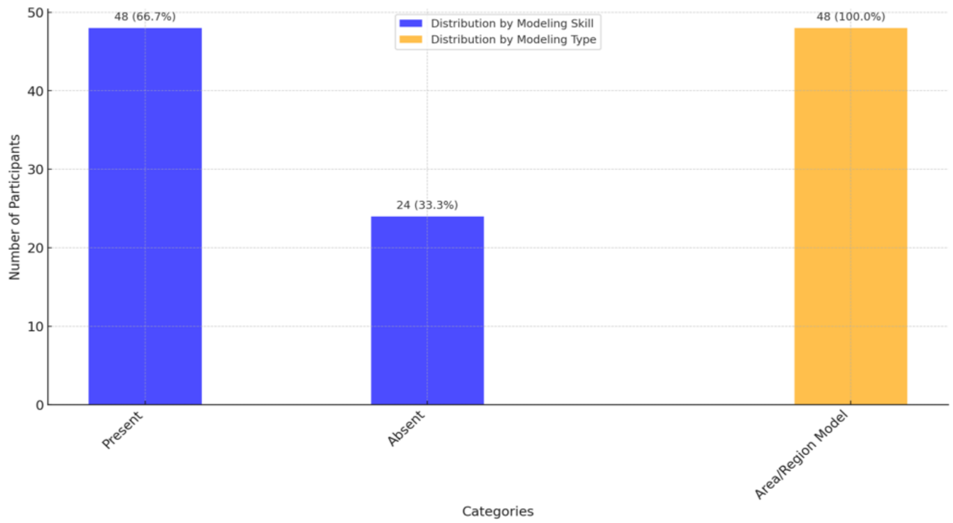
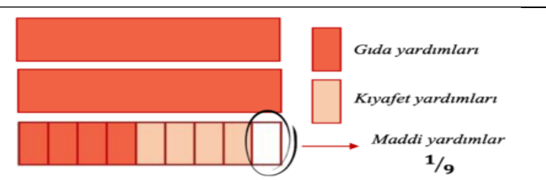
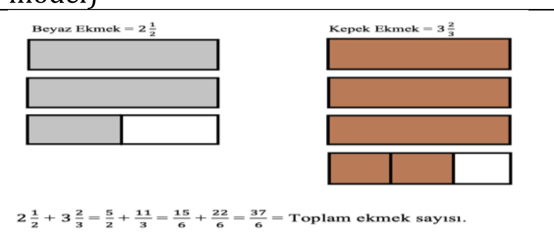
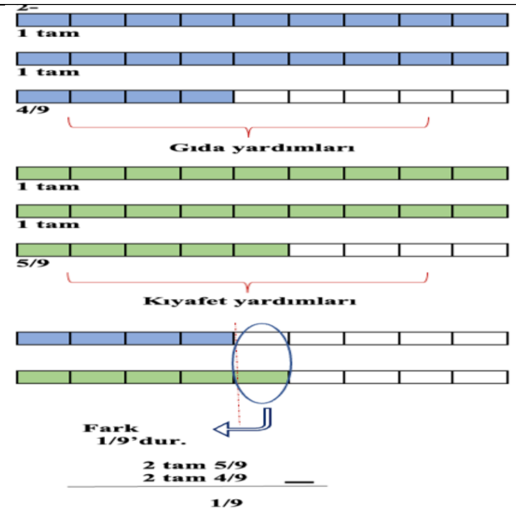
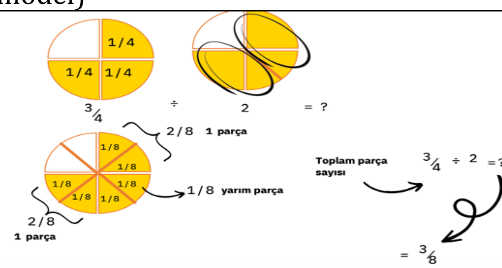
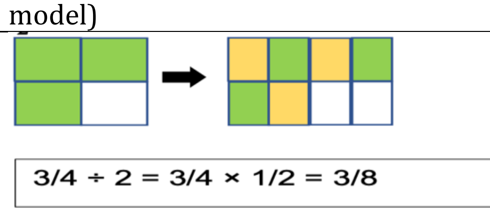


Figure 6: The modelling status and model type distributions of problems posed in the structured problem-posing situation

The distributions of the situations representing the problems posed based on their meanings in the structured problem-posing situation (Figure 5) and their modelling (Figure 6) are presented in Table 4.

Table 4: Examples of structured problem posing and modelling

The Meanings of the Four Operations	Examples of Structured Problem Posing Situation	Examples of Modelling
Separation Problems	"Donations are being collected under three categories for a charity campaign. Of all the donations, 4/9 are food, 4/9 are clothing and the remaining part is financial aid. How much financial aid is there?" (PPT 58)	 (Representation with the area/region model)
Joining Problems	"A bakery has 2 whole 1/2 loaves of white bread dough and 3 whole 2/3 loaves of whole wheat bread dough. How much bread dough is there in total?" (PPT 57)	 (Representation with the area/region model)

Comparison Problems	"Donations are being collected under two categories for a charity campaign. Of all the donations, 2 whole 4/9 are food aid, and 2 whole 5/9 are clothing aid. How much more is the clothing aid than the food aid?" (PPT 52)	 (Representation with the number line model)
Equal Groups Problems	"There is 3/4 of a tress leches cake on the counter. If Ela divides this cake into two equal parts, what fraction of the cake does each piece represent?" (PPT 59)	 (Representation with the area/region model)
Multiplicative Comparison Problems	"There is 3/4 of a tres leches cake on the counter. If my sister eats half of it, how much cake has she eaten?" (PPT 61)	 (Representation with the area/region model)
Inability to Pose Problems	Ability: Absent (PPT 54)	Ability: Absent

Preservice Primary Teachers' Views on Problem Posing and Modelling in The Subject of Fractions

After conducting the problem-posing and modelling activity in the subject of fractions, semi-structured interviews were held with the preservice primary teachers. The frequencies (f) of the themes and sub-themes generated through content analysis of the responses given by 32 primary teacher candidates to the interview questions are presented below.

Theme 1: Benefits of Problem Posing

The participants stated that they found the problem-posing activities beneficial (f=32). It was stated that the activities supported the development of the participants by improving their creativity and critical thinking (f=12), connecting mathematics with daily life (f=10) and contributing to an in-depth understanding of the subjects (f=10). Some examples from the statements of participants are as follows:

"We are confronted with problems in many parts of the daily life. Solving the questions we may encounter by turning them into a problem allows us to progress in a more systematic way." (PPT 25)

"Problem posing requires creativity and allows students to develop in this area." (PPT 10)

"By posing a problem, a person can also develop the ability to understand the problem. While teaching the relationship between daily life and mathematics, we prepare them for life through problems." (PPT 11)

Theme 2: Benefits of Problem Posing with Fractions

The participants stated that posing problems with fractions made various contributions (f=32). These contributions were expressed as the contributions of problem posing to the conceptual understanding of fractions (f=14), the elimination of deficiencies in this subject (f=10), and the development of teaching skills regarding fractions (f=8). Some examples from the statements of participants are as follows:

"The problem-posing activity related to fractions helped me better understand the logic of fractions." (PPT 32)

"I had quite a hard time making sense of some operations in fraction problems and realized my deficiencies." (PPT 16)

"Working with fraction problems was very valuable for my teaching methods. I learned how to concretize the topic." (PPT 12)

Theme 3: Problem Posing Type Preferences

Among the problem posing types, the participants mostly preferred the free problem posing situation (f=18) because they stated that they could use their creativity more freely in this type. While the existence of certain rules in problem posing for semi-structured and structured problem posing situations provided advantages for some participants, some stated that they felt limited (f=14). Some examples from the statements of participants are as follows:

"I felt more competent in free problem posing because I had to push myself to think more as I didn't have to commit to anything." (PPT 12)

"I felt more competent in the structured problem-posing type because certain rules and points to consider guided me." (PPT 20)

"I felt more competent in posing free problems because, in semi-structured and structured problems, there is some form of intervention, even if small. Our imagination is limited. However, in free problem posing, we can freely create a problem based entirely on our curiosity." (PPT 76)

Theme 4: Competences in Problem Posing with Fractions

The participants did not consider their problem-posing competences to be at a fully sufficient level (f=32). They stated that they needed to practice more (f=12). Some participants stated that they could be competent at a medium level (f=9). This medium level of competence was explained by feeling more competent in free problem-posing situations (f=11). Some examples from the statements of participants are as follows:

"I need to improve myself in problem posing because the problems I have posed consist of low-level problems." (PPT 18)

"My problem-posing skill is not very improved, but I think it is at a medium level." (PPT 24)

“I felt more creative and free in posing free problems.” (PPT 12)

Theme 5: Model Choice and Alternatives

The participants generally preferred the area/region model when modelling problems (f=32). It was stated that the area/region model was more understandable and easily applicable (f=14) and more suitable for modelling the problem (f=10). While some participants accepted that different models could be used, they stated that they preferred to use the first model that came to their mind (f=8). Some examples from the statements of participants are as follows:

“It was easier to design square or rectangular models in a computer environment.” (PPT 10)

“I took care to ensure that the models were appropriate for the problem, so I preferred the more commonly used models.” (PPT 12)

“I could have created more options for modelling problems, but I was limited by my lack of experience.” (PPT 27)

RESULTS AND DISCUSSION

In the current study, the problems posed by preservice primary teachers based on the four operations with fractions (free, semi-structured and structured) and their modelling skills in converting these problems into visual representations were examined.

The first result of the study is that when the problems posed by the participants in free problem-posing and semi-structured problem-posing situations are ranked from most to least based on the meanings of the four operations, in both categories, the first two positions are occupied by the problems with the meaning of separation in the subtraction operation and the meaning of equal groups in the division operation, respectively. In the case of structured problem posing, this ranking was simply reversed. In all the categories, apart from the problems occupying these two first positions, the other problems posed are mostly related to the joining meaning of the addition operation and the multiplicative comparison meaning of the multiplication operation. Since there is no research in the literature that specifically addresses this issue, the findings obtained in the current study should be considered a unique result for discussion. However, it can be said that the results of problem posing in subtraction operations expressed by some studies evaluating the types of operations (Akbaba-Dağ and Kılıç-Şahin, 2019; Akçay and Ardıç, 2020) are the opposite of what was found in the current study. According to them, the primary teacher candidates did not experience any problems in subtraction operations and were able to pose problems without errors. Similar to the finding of the current study indicating that the problems with the equal groups meaning in the division operation ranked second among the most frequently posed problems, Yao et al. (2021) found that preservice primary teachers were highly competent in posing division problems with fractions. The low rate of preference for problems involving multiplication can be explained by individuals’ difficulty in understanding the concept of multiplication (Doğan-Coşkun, 2019; Leung and Carbone, 2013; Lu, 2009; McAllister and Beaver 2012; Ünlü and Ertekin, 2012).

The participants who included in the study were able to pose problems in almost all types of problem-posing situations. It should also be emphasized that, contrary to the findings of Chapman (2012), the primary teacher candidates stated that they felt most comfortable and competent in free problem posing situations. However, Chapman (2012) reported that free problem-posing situations were challenging for them. In the study conducted by Kılıç (2013), the participants were found to be successful in posing free problems, as in the current study. However, they felt less competent in structured and semi-structured problem-posing situations compared to other types of problem posing.

The primary teacher candidates do not consider themselves fully competent in problem posing in the subject of fractions. They generally expressed that they felt the need to improve their skills in problem posing in the subject of fractions. Similar expressions were also uttered by the participants and researchers of the studies conducted by Canbazoğlu and Tarım (2021), Doğan-Coşkun (2019), Kılıç (2013; 2015), Lu (2009), and McAllister and Beaver (2012). A very small portion of participants - those who were more successful in free problem posing and preferred to pose such problems - felt more confident in posing problems involving fractions.

The second finding of the current study is that a significant portion of the preservice primary teachers were able to model the problems they posed in all the three problem posing categories. They were most able to model the problems they posed in the free problem posing situation. Structured problems were the second most posed problems, followed by semi-structured problems. In all the three problem posing situations, the area/region model was used the most. Although in small numbers, it can be seen that the set and number line models were also used. The participants almost exclusively preferred similar models while modelling the problems. Polat (2023) stated that there were difficulties in representing fractions with various models, which is consistent with the findings of the current study. The primary teacher candidates this situation as the ease of modelling and the selection of the most suitable model for the problem. They also emphasized that the selections were mainly based on their habits. Although they stated that alternative models could also be used during problem modelling, they were unable to reflect this in the examples they provided. Although modelling was performed in the current study, the inability to do so is parallel to McAllister and Beaver (2012), where the participants experienced shortcomings in modelling the processes of problem posing with fractions and correctly expressing the contextual units.

The preservice primary teachers generally described problem posing as a beneficial activity. The current study revealed that they gained a better understanding of the concept of fractions through problem-posing activities. The findings showed that problem-posing activities had a positive impact on the participants' mathematical knowledge and their views on mathematics (Toluk-Uçar, 2009). They reported that problem-posing activities led to improvements in themselves as learners. They also stated that problem posing helped them to develop skills such as creativity and critical thinking, and made it easier to establish a connection between real life and mathematics. Rivzi (2004) states that problem-posing activities strengthen the connections made with real life and that problem posing skills foster creativity and flexibility. Chapman (2012) states that problem posing is an effective method for developing primary teacher candidates' mathematical thinking skills and teaching capacities, and that various types of problem posing allow participants to think creatively and flexibly in different mathematical contexts. Li et al. (2020) observed that preservice primary teachers developed their mathematical thinking, problem-solving and creativity skills in posing problems with fractions.

The preservice primary teachers stated that the problem-posing process, specifically related to the concept of fractions, helped them understand the concept of fractions better and made them aware of their personal misunderstandings regarding fractions. Chapman (2012), Yao et al. (2021) and Xie and Masingila (2017) stated that problem-posing activities involving fractions help primary teacher candidates gain a deeper understanding of mathematical concepts. Moreover, it was confirmed that fraction-related problem-posing activities would be an effective tool for measuring the deficiencies in this area (Kılıç, 2015; Leung and Carbone, 2013). In addition, it is evident that they gained the necessary knowledge about the concept of fractions to teach it in their future active professional careers. Li et al. (2020) found that teacher candidates who participated in a problem posing-focused professional development program showed significant improvement in their problem-posing skills and their belief in the benefits of this method. They became more prepared to integrate problem-posing activities into their classrooms more effectively.

RECOMMENDATIONS

In light of the results of the current study, it can be recommended that, for the integration of problem-posing activities into teacher training programs, these activities should be emphasized more to help the preservice primary teachers develop their understanding of the concept of fractions and their skills in the four basic operations. It is also recommended that these activities be systematically included in curricula, offered in various formats such as structured, semi-structured, and free types, and enriched with different contexts and interdisciplinary approaches. In the study, all the problems posed in the free and semi-structured problem posing situations are directed towards the result unknown. In a future study, problem posing situations involving different problem structures (result unknown, change unknown and start unknown) could be included. In addition, problem posing-focused professional development programs could be organized for them. These programs should focus on enhancing both their problem posing skills and beliefs in the educational benefits of this method.

To emphasize problem variety and the meanings of operations, greater focus should be placed on problem-posing activities related to multiplication and addition to help the primary teacher candidates understand the meanings of the four operations. In particular, they should be encouraged to pose contextual and real-life-based problems for these operations. To address this situation, instructional strategies incorporating conceptual cues and guidance can also be utilized.

For a better understanding of the concept of fractions in terms of concretization and model usage, it is necessary to diversify the use of concrete models. Encouraging educational materials should be prepared to motivate preservice primary teachers to use alternative model options during problem modelling. Fo r example, digital tools and technological platforms can be used to further support their problem-posing and modelling skills. Visual representations and interactive applications related to fractions can facilitate the problem-posing process.

In addition to all these, long-term studies can be conducted to examine the impact of problem-posing and modelling activities on the preservice primary teachers' pedagogical and mathematical competences. These studies can also assess the level of retention of learning from activities and their indirect effects on students.

AUTHORS' CONTRIBUTION

EE contributed to the writing of the manuscript, data collection, data analysis, and language editing. Dr. NIT contributed to identifying the research topic, data analysis, data collection, language editing, and the revision process.

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APPENDIX 1: SEMI-STRUCTURED INTERVIEW FORM

1. Demographic Information (Please complete this section thoroughly.)

Name of the University you are attending: Gender:

2. Interview Questions (This section contains questions related to the purpose of the study. Please ensure your answers are detailed and explanatory.)

a) Do you think problem-posing activities in mathematics classes are beneficial? Please explain.

b) Do you believe that problem-posing activities related to fractions contribute to your learning/teaching of fractions? Please explain.

c) In which type of problem-posing activity did you feel more competent? Please explain.

d) If you were asked to evaluate your problem-posing skills with fractions, what would you say about your competence? Please explain.

e) Why did you choose the models you used while modeling the problems? Could other models have been used? Please explain.