Pak. j. life soc. Sci. (2025), 23(1): 7383-7401 E-ISSN: 2221-7630;P-ISSN: 1727-4915

Pakistan Journal of Life and Social Sciences

www.pjlss.edu.pk

https://doi.org/10.57239/PJLSS-2025-23.1.00574

RESEARCH ARTICLE

Influence of the 8Ps Learning Model on Grade 12 Students' Performance in Differential Calculus: A Document-Analysis Approach

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ARTICLE INFO

Received: Jan 7, 2025

Accepted: Feb 21, 2025

Keywords

Document Analysis

8Ps Learning Model

Stationary Points in Differential Calculus

Mathematical Problem-Solving Skills

Student Performance

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ABSTRACT

The persistent difficulties that grade 12 students experience with intricate mathematics topics like differential calculus necessitate exploring effective instructional strategies to support their learning. This qualitative study adopted a document-analysis approach to determine how the 8Ps learning model can influence student performance by examining post-test scripts from 238 grade 12 students based on stationary points differential calculus. Building on our previous research, which utilized classroom observations, descriptive statistics and independent samples t-test to suggest the potential of the model to strengthen students' mathematical problem-solving skills, the present study sought to identify the underlying reasons for the students' solutions and pinpoint areas needing improvement. For data collection, we developed a seven-criterion, mathematical problem-solving assessment form (MPSAF). This form evaluated how skillfully participants interpreted mathematical problems, incorporated prior knowledge and applied appropriate formulas. It also appraised how properly they provided and justified solution-steps and applied their solutions to understand related mathematics tasks. The criteria established in the MPSAF were deployed as themes for data analysis. The findings signify that the experimental group, which received the 8Ps-oriented instruction, portrayed higher levels of reasoning and more logical mathematical problem-solving strategies compared to the control group that was traditionally instructed, thus corroborating our prior study.

INTRODUCTION

As a fundamental branch of mathematics that is integral to most other divisions of mathematics, differential calculus underpins various fields which include physics, engineering, economics and social sciences (Feudel & Biehler, 2021; Nuñez et al, 2023). Its principles are essential for solving real-world problems, such as resource optimization and dynamic system modeling (Kafunga, 2024). Its interconnectedness with other areas of mathematics and valuable uses across diverse disciplines underline its relevance in contemporary education and professional practice (Collins, 2022; Hagjoo & Reyhani, 2021). Despite its broad applications, 12th graders around the world struggle with understanding differential calculus concepts (Estonanto & Dio, 2019; Nuñez et al., 2023). Research reveals that the students' difficulties stem from factors such as the abstract nature of calculus (Auxtero & Callaman, 2020; Kafunga, 2024), inadequate foundational knowledge in prerequisite topics such as functions and algebra (Jaudinez, 2019; Wewe, 2020), and ineffective instructional strategies that do not engage students in meaningful learning experiences (Makgakga & Makwakwa, 2016; Simovwe, 2020). Moreover, students' perception of differential calculus as a complex and daunting subject further deters them from mastering it (Collins, 2022; Kafunga, 2024).

As an intervention strategy, we constructed the 8Ps learning model and, in the current study, employed a document-analysis technique to explore the model's influence on grade 12 students' performance in stationary points within differential calculus. From the account of Merriam and Tisdell (2016), document analysis is a qualitative research method that thoroughly reviews and interprets documents to extract relevant insights that align with a specific research focus. This type of analysis can cover a wide array of documents such as: reports, written texts, transcripts, and other forms of recorded communication. The intent is to understand the context, meaning and implications of the documents under review, often by examining their content, structure and intended purpose. As noted by Morgan (2022), this approach allows for systematic analysis of educational materials and student outputs to identify patterns and insights.

Effective problem-solving strategies are crucial for mastering challenging concepts like differential calculus (Nuñez et al., 2023). Traditional teaching methods often fall short in equipping students with the necessary skills to approach mathematical problems with confidence (Simovwe, 2020). With students passively receiving information from the teacher, learning by rote memorization, and studying differential calculus concepts in isolation, traditional teaching methods have resulted in low academic performance and limited knowledge transfer among students (Mendezabal & Tindowen, 2018; Yimer & Feza, 2019). Consequently, the teaching approaches typically render students unable to apply their differential calculus skills meaningfully in real-life contexts (Panero, 2024; Vacalares et al., 2024). Therefore, exploring alternative pedagogical approaches becomes necessary (Bedada, 2021; Klang et al., 2021). One such innovative instructional model that has emerged to address the concern is the 8Ps learning model, designed to provide a comprehensive framework aimed at fostering deeper engagement with complex mathematical topics such as differential calculus.

The current study was executed as a follow-up to our previous research which adopted classroom observations, descriptive statistics and independent samples t-test to suggest that the 8Ps learning model can support grade 12 students' mathematical problem-solving skills, particularly in stationary points differential calculus. The current study adopted document analysis to evaluate how the 8Ps learning model impacted students' performance in this vital mathematical area. We analyzed only the post-test scripts of 238 participants from that prior study (leaving out their pre-test scripts) since our focus was essentially to uncover the underlying reasons behind the participants' solutions to the mathematics tasks assigned them (consequent upon implementing the 8Ps model) and to determine the specific aspects of their problem-solving skills needing further development.

To this end, we designed a mathematical problem-solving assessment form, MPSAF (Appendix A1), which sought to evaluate participants' post-test scripts based on the skill level depicted in terms of: representing questions as patterns that may aid solving them; interpreting and translating assigned questions into solvable forms; connecting questions to their prior knowledge as a clue to the required solutions; applying proper principles and formulas; showing logical and sequential solution-steps; providing appropriate justifications for solution-steps; and explaining as well as applying solutions to understand related questions.

Despite the undeniable value of the document analysis in research, it has recorded a limited use in the literature for evaluating grade 12 students' mathematical problem-solving skills specifically within the context of stationary points in differential calculus (Morgan, 2022; Tight, 2019). To address this gap, the present study sought to contribute insightful information on the application of a document-analysis method to measure the effect of heuristic learning models such as the 8Ps model on students' mathematics learning outcomes. We assert that this research is capable of guiding future pedagogical practices and enhancing the teaching and learning of differential calculus and broader mathematics in secondary education.

STUDY CONTEXT

A brief retrospective is included to contextualize the current study. The overarching objective of the previous research was to develop productive mathematical problem-solving strategies for grade 12 students to improve their performance in mathematics, especially in the concept of stationary points within differential calculus. This brought about why we created the 8Ps learning model, which construction was reinforced by Pólya (1945) problem-solving model as shown by Figure 1.

Construction of the 8Ps Learning Model Reinforced by Pólya's Model

The 8Ps learning model builds on the foundation of Pólya's influential framework outlined in his 1945 work, *How to Solve It*. Pólya (1945) identifies four key problem-solving stages: *understanding the problem, devising a plan, carrying out the plan*, and *looking back*. These stages emphasize the importance of comprehension, strategic planning, execution, and evaluation in mathematical problem solving (Chacón-Castro et al., 2023). The 8Ps model expands on this by providing four additional phases for a comprehensive understanding of the problem-solving process as illustrated in Figure 1. In the 8Ps model, *understanding the problem* is split as *probing* and *pinpointing*, focusing on thoroughly defining a problem and identifying its critical elements. This idea mirrors *entry*, the first of the seven problem-solving stages by Mason et al. (1982/2010). It also incorporates techniques from Maccini and Gagnon (2006) as well as Cherry (2011).

In the second step, *devising a plan*, the 8Ps model introduces *patterning* and *projecting*. While *patterning* involves creating mathematical representations, *projecting* is about developing solution strategies (Kirkley, 2003; Maccini & Gagnon, 2006). Pólya's third step, *carrying out the plan*, is represented as *prioritizing* and *processing* in the 8Ps model. *Prioritizing* entails selecting the most relevant of the solution plans projected, while *processing* executes these plans methodically. Lastly, Pólya's *looking back* is reflected in the 8Ps model as *proving* and *predicting*, where solutions are assessed for correctness (during *proving*) and applicability to similar problems is considered (when *predicting*). This idea tallies with Burton (1984) who describes stage four of his problem-solving process as *extension*. The model's eighth phase encourages students to leverage their current understanding to predict solutions to future problems, extending beyond Pólya's fourth step, *looking back,* which primarily reflects on the past solution strategies taken without explicitly promoting future predictions.

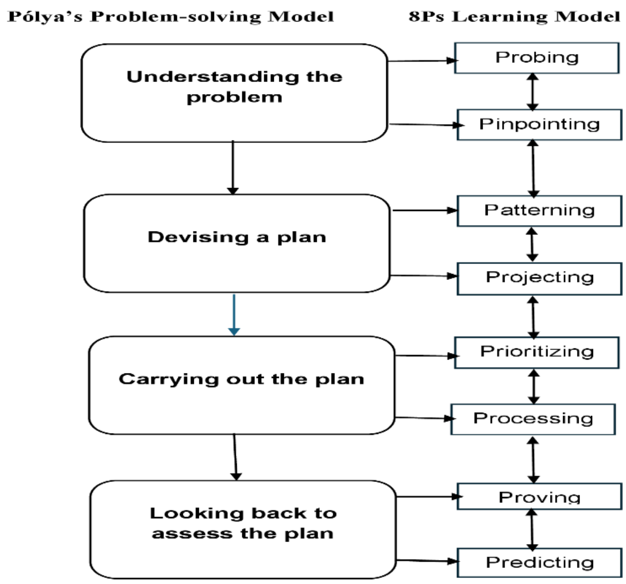


Figure 1. The Construction of the 8Ps Learning Model as Inspired by Pólya's (1945)

The design of the 8Ps learning model underscores strategic thinking and reflection during problem solving. Again, it accepts that not all phases may apply to every problem, thus allowing for flexibility in the problem-solving process. This adaptability encourages students to revisit earlier phases when need be. In the problem-solving process, the teacher's main duty is facilitation, promoting student collaboration and peer learning. We argue that the 8Ps model's multiple phases are more of an advantage than a drawback. The multiple phases present a range of interconnected approaches to tackling problems, fostering a deeper comprehension of mathematical concepts among students. By deliberately engaging with each phase, students are prompted to investigate problems comprehensively, consider diverse viewpoints, and actively develop their understanding. Consequently, the model supports critical thinking and offers a systematic approach to solving challenging mathematical problems, equipping students with versatile problem-solving skills.

Implementation of the 8Ps Learning Model in the Previous Research

The 8Ps learning model was implemented in our prior research – a quasi-experimental study with pre-test/post-test, non-equivalent, control group. Participants in the study were 253 grade 12 mathematics students and eight grade 12 mathematics teachers from eight high schools of an education district in Gauteng province. While the selection of the education district was purposive because the district possessed the characteristics that suited the objectives of the study, the eight schools were conveniently sampled mainly because of their accessibility and availability for the study (Andrade, 2021). Four of the schools (consisting of 128 students and 4 teachers altogether) formed the experimental group and the other four (having a total of 125 students and 4 teachers) constituted the control group.

Since quasi-experimental research, characteristically, estimates causal relationships without randomization of subjects (Rogers & Révész, 2020; Truong Tran Minh & Mahmood, 2024), neither the schools nor the participants were randomized into groups. Only one intact classroom of students per school was considered. The schools in the experimental group were kept reasonably apart from those in the control group to avoid compromising the research results (Em, 2024; Gopalan et al., 2020). To uphold ethical standards, the unique code identifiers EX1 to EX4, EX-T001 to EX-T004 and EX001 to EXL128 were assigned accordingly to the schools, teachers and students in the experimental group. The control group was similarly issued the identifier codes CL1 to CL4, CL-T001 to CL-T004 and CL001 to CL125. Kang and Hwang (2023) recommend that the use of an identification system, as demonstrated here, serves as an ethical safeguard for protecting participant privacy during data collection, analysis and reporting processes, ensuring the integrity of the research.

In the experimental group, the primary researcher (PR) administered the 8Ps intervention lessons by himself to make sure that the instruction was thoroughly, consistently and comprehensively delivered. This approach was adopted for the actual effect of the learning method to be realizable (Dhlamini & Mogari, 2011; Gay et al., 2012; Masilo, 2018; Ofori-Kusi, 2017). In contrast, the control group's four regular mathematics teachers taught the same series of lessons on stationary points in differential calculus through traditional teaching methods. To forestall potential researcher biases, the PR was guided by the literature in acting as the intervention teacher. Although the four regular mathematics teachers at the experimental schools did not directly carry out the intervention, each of them officially observed two intervention lessons and provided feedback which enriched the analysis of the emergent data for that research. Their presence also contributed considerably to maintaining classroom order and engaging the students, as the students perceived their teachers' support for the research. The researcher only paid pre-arranged visits to the control group schools to observe the teachers' lessons as a non-participant, attending four lessons in each school. From these observations, the PR gathered that the traditional methods employed by the teachers differ in nature and mode of presentation from the 8Ps learning model applied in the experimental group.

During the two-month investigation, participants were required to complete a problem-solving achievement test twice. The first instance was a pre-test written by 253 participants in the first week, prior to the 8Ps-oriented intervention. The second instance was a post-test conducted in the last week, after the intervention. Only 238 of the participants took part in the post-test because 15 of them (9 from the experimental group and 6 from the control group) could not make it to the post-test stage. This clarifies why only 238 post-test scripts were analyzed in the current study. The essence of the achievement test was to evaluate the influence of the 8Ps learning model on the participants' mathematical problem-solving performance. The pre-test measured the participants' initial knowledge of the mathematical concept, while the post-test evaluated any possible improvement in their skills post-intervention. The test was based on stationary points in differential calculus, a crucial topic in South Africa's grade 12 mathematics curriculum, which accounts for 35 ± 3 marks of the 150 marks allotted to Paper 1of the NSC mathematics examination. The test, which consisted of five major questions, were designed to prompt the participants to engage in significant mathematical thinking and apply various solution strategies. The duration of each test was 90 minutes, and the total marks was 85, which was later converted to percentage.

Although the test was drawn from the 2017 - 2019 DBE/NSC examination questions on stationary points in differential calculus, which were already constructed in compliance with the necessary assessment guidelines in the grade 12 mathematics Curriculum Assessment Policy Statement (CAPS), it was still subjected to validity and reliability processes. Four experts in mathematics education – a seasoned Department of Basic Education (DBE) Subject Advisor for mathematics in the Further Education and Training (FET/grade 10-12) phase and three experienced teachers who had taught grade 12 mathematics for about twenty years each – assisted with the content and construct validity of the test. Reliability was established by pilot-running the test on 82 mathematics students at a secondary school which did not partake in the actual study. The test-retest correlation coefficient obtained after two weeks was 0.87, and the average Cronbach's Alpha coefficient was 0.698 (approximately 0.7), indicating sufficient internal consistency reliability (Zakariya, 2022). This suggests that the achievement test could be appropriate for the main study.

After administering the pre-test and post-test in the main study (that is, the prior study), participants' marks were analyzed with the descriptive statistics and independent samples t-test. Findings had reported that the 8Ps learning model may support students' mathematical problem-solving skills, particularly in stationary points differential calculus. The current study subsequently saw the need to further investigate this finding by applying a different research method. This study noted particularly that, in the context of evaluating grade 12 students' problem-solving skills in stationary points differential calculus, not much has been recorded in the literature on the use of document analysis (Moilanen et al., 2022; Morgan, 2022; Tight, 2019). This gap presents an opportunity for the current study to contribute to this research area.

Problem Statement of the Present Study

Differential calculus is a fundamental mathematics field with significant applications in various disciplines and real-world context, promoting critical thinking and problem-solving skills (Nortvedt & Siqveland, 2018; Nuñez et al., 2023; Yimer & Feza, 2019). However, it remains a challenging topic for grade 12 students worldwide, South Africa inclusive (Dreyfus et al., 2021; Tasara, 2022; Sebsibe, 2019). Traditional teaching methods often focus on rote learning and passive knowledge transfer, which hinder meaningful engagement and desirable student performance (Bedada, 2021; Dhlamini, 2012; Kafunga, 2024). To address this concern, the current study introduced the 8Ps learning model, designed to foster active student involvement and structured learning. It sought to assess the model's impact on student performance in differential calculus by analyzing participants' post-test scripts to understand their reasoning and solution strategies across the model's eight phases, ultimately identifying areas for targeted skill development.

Research Questions

The main research question addressed in this study to attain the set goal is stated as follows:

- *What is the mathematical reasoning behind grade 12 students' solutions to questions on stationary points in differential calculus following the 8Ps-based intervention?*

For a clear and comprehensive answer to this question, the following sub-questions were considered through the lens of the seven MPSAF metrics:

Following implementing the 8Ps intervention based on stationary points in differential calculus,

- *What level of mathematical reasoning do the students demonstrate in representing questions as patterns that may aid solving them?*
- *What level of mathematical reasoning do the students demonstrate in interpreting and translating questions into solvable forms?*

- *What level of mathematical reasoning do the students demonstrate in connecting questions to their prior knowledge as a clue to the required solutions?*
- *What level of mathematical reasoning do the students demonstrate in applying correct formulas and principles to solve questions?*
- *What level of mathematical reasoning do the students demonstrate in showing logical and sequential solution-steps for assigned questions?*
- *What level of mathematical reasoning do the students demonstrate in providing appropriate justifications for their solution-steps?*
- *What level of mathematical reasoning do the students demonstrate in explaining and applying their solutions to understand related questions?*

Theoretical Structure

The study is rooted in the cognitive, problem-solving theories of John Dewey and Graham Wallas. Arguing that effective problem-solving process hinges on conscious thought and deliberate action, Dewey (1910/1938) suggests five mental steps*: identifying the problem, diagnosing it, gathering data to form a hypothesis, testing that hypothesis,* and *drawing conclusion* (Brunning et al., 1999; Kulsum & Kristayulita, 2019; Williams, 2017). This framework aligns with the main aim of the present study which, basically, is to understand the students' reasoning that produced their solutions to assigned mathematics tasks and identify those areas of their mathematical problem-solving skills needing improvement. Dewey's cognitive steps also correlate with the seven standards set by the MPSAF. The eight phases of the 8Ps learning model also resonate with Dewey's cognitive stages.

In his own case, Wallas (1926) recommends four cognitive processes: *preparation*, *incubation*, *inspiration*, and *verification.* At *preparation*, the problem is analyzed, and necessary information gathered. *Incubation* allows for subconscious processing of the problem as it is not actively focused or considered at this point. *Inspiration* occurs when a solution emerges from this subconscious effort, while *verification* entails evaluating the solution's validity (Andre, 1986; Salvi et al., 2016; Setyana et al., 2019; Wallas, 1926). Wallas' stages equally complement the MPSAF parameters, providing further insights into how to utilize the MPSAF to understand the students' problem-solving approaches. *Preparation* clarifies initial strategies used by the students; *incubation* highlights their reflective thinking processes; *inspiration* identifies moments of sudden insight, particularly among those participants exposed to the 8Ps model, and *verification* evaluates their ability to justify their solutions. Overall, Dewey's and Wallas' cognitive principles substantially inform the analytical approach considered in this study. Figure 2 presents an overview of this.

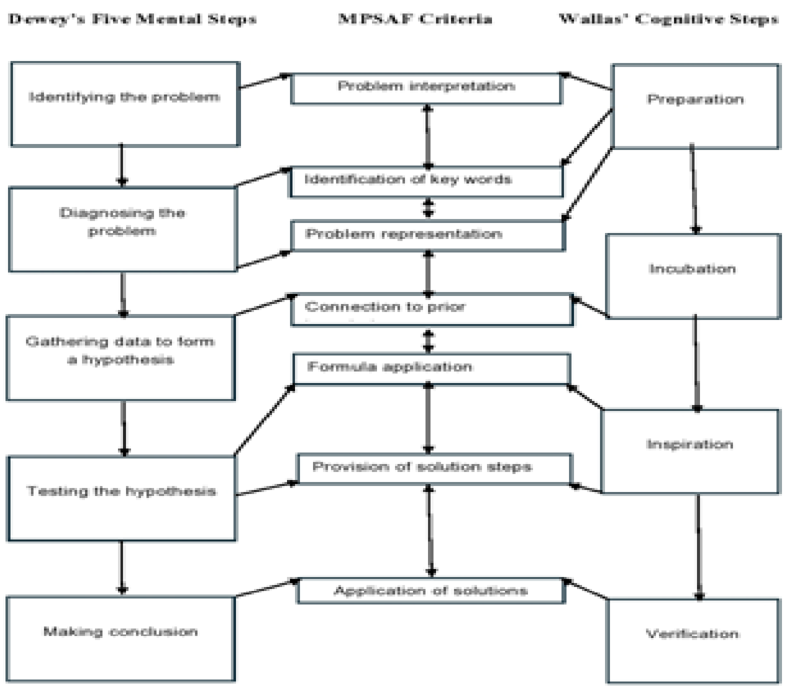


Figure 2

Theoretical Structure Synthesizing Dewey's and Wallas' Cognitive Steps to Reinforce the Mathematical Problem-solving Assessment Form (MPSAF)

LITERATURE REVIEW

Document analysis is a qualitative method in research focusing on the systematic evaluation of printed, digital, or physical documents to uncover meaning and draw empirical conclusions (Drisko & Maschi, 2016). It assesses private documents, institutional records, and artifacts for exploratory or explanatory analysis (Morgan, 2022). It categorizes content into thematic patterns or uses structured rubrics to obtain deep insights (Drisko & Maschi, 2016). The goal of document analysis is to understand the context, significance, and implications of the documents being studied. This can involve examining the content, structure, and purpose of the documents (Prior, 2014).

Document analysis can be a stand-alone approach or combined with other qualitative and quantitative methods for triangulation. It offers rich data often overlooked due to limited awareness of its potential (Merriam & Tisdell, 2016; Prior, 2014). It reveals hidden meanings and nuances not easily uncovered by interviews or observations, and its non-intrusive nature allows examination of pre-existing documents without influencing participants (Morgan, 2022). It captures both explicit and latent meanings for a detailed understanding (Braun & Clarke, 2013).

Nelson and Carter (2022) conducted a qualitative document analysis of early mathematics interventions, focusing on how interventions support vocabulary learning in mathematics. The study found that targeted vocabulary instruction significantly improved students' mathematical understanding and problem-solving capabilities. The study suggested incorporating vocabulary strategies into an instructional model to enhance students' ability to interpret and solve mathematical problems effectively. The eight phases of 8Ps learning model can serve as helpful vocabulary strategies. Morgan (2022) discussed qualitative document-analysis techniques to extract meaningful insights from educational documents. Findings emphasize the importance of rigorous document analysis in educational research to derive valid conclusions about teaching effectiveness, while also highlighting its value in evaluating student performance through post-test scripts.

Thomas and Dyches (2019) performed a critical document analysis, examining the hidden curriculum within a reading intervention program. Findings revealed implicit biases and assumptions in the intervention that could affect student engagement and learning outcomes. The study highlights the importance of transparency in instructional models like the 8Ps to ensure equitable learning opportunities in mathematics. Elo et al. (2014) in Finland focused on qualitative content analysis trustworthiness to ascertain reliability and validity in research findings. Findings outline best practices for conducting qualitative content analyses to enhance trustworthiness. To this end, the present study provides a structured and validated MPSAF framework for ensuring a rigorous analysis of participants' post-test scripts within the context of the 8Ps learning model. The reviewed studies collectively underscore the significance of structured instructional models like the 8Ps and the MPSAF in enhancing students' mathematical problem-solving skills. They emphasize various strategies – such as vocabulary integration, transparency in teaching methods, and systematic document analysis – that may suggest the potential of the 8Ps model to address persistent challenges in learning complex mathematical concepts like differential calculus.

METHODOLOGY

Study Design

This investigation applied an interpretive design to explore participants' meaning-making processes and how the 8Ps model influenced their reasoning and problem-solving strategies. The inherent flexibility of this design facilitated the acquisition of rich qualitative data from the post-test scripts, resulting in a deep analysis of participants' thought processes and solution strategies.

Unit of Analysis

A total of 238 post-test scripts were analyzed – 119 from the experimental group and 119 from the control group.

Instrumentation and Validation

This study utilized a researcher-created mathematical problem-solving assessment form (MPSAF) benchmarked against relevant literature and existing high school mathematics rubrics to ensure that its construction adhered to standard practices. To establish its face and content validity, the MPSA underwent a thorough review by three seasoned high school mathematics teachers, each with over ten years of experience teaching mathematics in grade 12. For reliability verification, each of the three teachers adopted the MPSAF to evaluate 30 test scripts from a previous term's joint grade 12 mathematics examination in their respective schools. The resulting Cronbach's Alpha coefficient of 0.77 indicates an acceptable level of internal consistency reliability for the MPSAF. This value also suggests that the seven yardsticks of the MPSAF were reasonably correlated and consistently measured the same underlying construct. This supported the suitability of the measuring tool for use in this study (Ghazali, 2016; Hajjar, 2018).

Ethical Considerations

As necessitated by research ethics (Connelly, 2014; Kumar, 2018), approval letters were obtained from the UNISA ISTE Ethics Review Committee and Gauteng Department of Education (GDE) to carry out the investigation from which the post-test scripts emanated. Participant written informed consent was also secured from participating schools, teachers, parents and students. Participants were fully informed about the voluntary nature of their involvements in the study, measures for protecting them from harm, and their right to withdraw at any time without consequences. Confidentiality was ensured by using arbitrary codes for analysis and reporting, which is why the analyzed post-test scripts contained code identifiers instead of participants' names.

Procedure for Analyzing the Post-test Scripts

The seven MPSAF measures were deployed as the themes upon which the analysis of the 238 post-test scripts was based. The post-test had five major questions with each question having sub-questions, giving 23 sub-questions in all (see Appendix A2). For each sub-question, the seven criteria of the MPSAF were applied. Evidence of a skillful use of a criterion was awarded 1 mark; thus, each sub-question attracted a maximum of 7 marks and a script a maximum of 161 marks (23 sub-questions multiplied by 7 marks each). Hence, it was possible to obtain the number of scripts/participants per group for each of the seven MPSAF performance areas. For this study, three performance categories were considered: low-level problem-solving performance (LL) = Below 60 marks, middle-level problem-solving performance (ML) = 60 - 90 marks, and high-level problem-solving performance (HL) = 90 or more. The cut-off marks were not made higher than that considering that students generally find mathematics challenging, particularly differential calculus.

As interpreted in this study, a script that showcased evidence of mastery of a skill was rated HL; one showing progress towards mastery was graded ML, and a script operating at a beginner's level of a skill application was classified LL. This procedure was consistently adhered to across all the post-test scripts. By leveraging the MPSAF as an evaluation tool, this study departs from the conventional assessment system, which, characteristically, concentrates on the accuracy of the question's solution as opposed to the processes or strategies involved in solving it. Scholars (e.g. Auxtero & Callaman, 2020; Horn & van Niekerk, 2020) argue that a well-crafted performance rubric (termed MPSAF in this study) offers teachers authentic information on student performance with respect to specific yardsticks, knowledge and processes. This enables teachers to give reliable scores that reflect students' progress against defined standards. In support, Rosli et al. (2013) describe performance rubric as comprehensive assessment tools for measuring students' learning, with specific guidelines that accord priority to the process of finding the solution, ultimately portraying the students' conceptual understanding of the given task.

Mitigating Potential Biases in the Analysis

To forestall possible variables that could skew the research findings, some specific steps were taken. First, the post-test scripts underwent independent evaluation by three graders, who adhered strictly to the seven metrics outlined in the MPSAF. The graders comprised the primary researcher, an experienced teacher who contributed to reviewing the MPSAF, and a trained research assistant with a background in mathematics education. Prior to the evaluation process, three sets of copies of the 238 scripts were prepared and distributed to each grader receiving one set. A calibration session was held where the three graders collectively reviewed five common scripts together, discussed their scoring, and reached a consensus on how to apply the MPSAF criteria effectively. Following this collaborative session, each grader proceeded to evaluating the post-test scripts independently. The Fleiss Kappa inter-rater reliability (IRR) measure resulting from the three sets of scores was 0.81, indicating a high level of agreement among the graders. Fleiss Kappa IRR, an extension of Cohen's Kappa IRR, is a robust approach for measuring agreement in multiple-grader evaluations involving categorical ratings (McHugh, 2012; Moons & Vandervieren, 2023). Despite attaining agreement across the score sets, their averages were computed and used as the final scores for analysis. The subsequent section details the analysis of these scores.

RESULTS

1. Representation of Mathematical Problems as Helpful Patterns

Heuristic problem-solving models generally have long recognized the cognitive advantages of representing mathematical problems in visual forms such as charts, tables, pictures, diagrams and maps. These representations or patterns can help problem solvers identify key relationships, recognize underlying structures, and develop more effective solution strategies (Kaitera & Harmoinen, 2022). Appreciating the value of visual representations, the MPSAF probed this crucial area of mathematical problem solving. From question 1 where $f(x) = 2x^3 - 5x^2 + 4x$, for instance, a problem solver may come up with the following tables as helpful patterns that may facilitate solving it.

Table 1: Mathematical Representations Formed from the Question as a Clue to Solution Strategies

Relevant hints	$\boldsymbol{f(x) = 2x^3 - 5x^2 + 4x}$
Degree of $\boldsymbol{f}$	3
Function type	cubic
Factorized form	$(2x - a)(x - a)$
Number of x-intercepts	3 (since it touches the x-axis at three points)
Number of y-intercepts	1 (since it touches the y-axis at one point)
Number of turning points	2 (since it is a cubic function)

Ascertaining the nature of the two turning points can be done as follows by examining their respective concavities:

Table 2: Another Mathematical Representations Formed as a Clue to Solution Strategies

Minimum turning point	Maximum turning point
Concave downwards	Concave upwards
U shape	∩ shape
$\boldsymbol{f''(x) > 0}$	$f''(x) < 0$
$\boldsymbol{f'(x) = 0}$	$f'(x) = 0$

A problem solver may find the two tables (representations) useful as hints for solving question 1.1 (which asked for the coordinates of the turning points of the graph of f); question 1.4 (that required the values of x for which the graph be concave up; question 2.5 (requiring the values of x for which $f''(x) < 0$) which directly was about upward concavity; question 3.2 (centered on maximum turning point).

Sub-RQ 1: *What level of mathematical reasoning do the students demonstrate in representing questions as patterns that may aid solving them?*

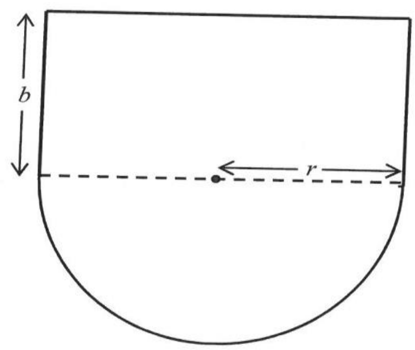
Table 3: Level of Participants' skillful Representation of Problems as Helpful Patterns

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	36.1% (43)	17.6% (21)	46.2% (55)
	Control	5.9% (7)	33.6% (40)	64.7%(75)

Based on Table 3, 36.1% of the experimental group participants attained a high level of skill here, which is about six times the 5.9% rate recorded by the control group. Besides, the ratio of the low problem solvers(LL) of experimental and control groups in this aspect is roughly 5:7 (i.e. 21:55). Only the medium-level performers (ML) of the control group were about double that of the experimental group. Regardless of that, the experimental group, on average, exhibited a significantly higher competency level in representing assigned mathematical problems as relevant patterns than the control group. This has provided an answer to sub-research question 1.

2. Translation of Mathematical Problems to Solvable Forms

The concept of stationary points in differential calculus not only demands that students possess the skill to successfully translate mathematical problems to solvable equations or forms, but also expects them to ably interpret the equations logically (Panero, 2015). Thus, the achievement test included questions that aimed to measure the participants' skills in both areas. Question 4.1 to 4.6 and 5.3 serve as good examples. In question 4, the participants were presented with a diagram of a theatre stage, which was a rectangle mounted on a semi-circle, with the length of the rectangle equal to the diameter of the semicircle. The perimeter of the theatre stage was given as 60 m. The participants' ability to solve the six sub-questions 4.1 to 4.6 hinged on their capacity to interpret the diagram and translate the questions into solvable equations. For example, question 4.1 asked them to determine the expression for b in terms of r, while question 4.2 required them to find the value(s) of r for which the area of the theatre stage would be a maximum, given that the function $f(x) = 3x^3$. A model translation of questions 4.1 and 4.2 into solvable equations is as follows:



(4.1) Perimeter = 2r + b+ b + half of circumference

$$60 = 2r + 2b + \frac{2\pi r}{2}$$

$$60 = 2r + 2b + \pi r$$

$$2b = 60 - 2r - \pi r$$

$$2b = 30 - r - \frac{\pi r}{2}$$

(4.2) Area of the theatre stage = Area of rectangle + Area of semi-circle

Area of the theatre stage = $(l \times b)$+ πr^2 = $(2r \times b)$+ πr^2 ....

Table 2 provides the analysis of the achievement of both groups in this performance category.

Sub-RQ 2: *What level of mathematical reasoning do the students demonstrate in interpreting and translating questions into solvable forms?*

Table 4: Level of Participants' Skillful Interpretation and Translation of Problems into Solvable Equations

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	37.8% (45)	16.8% (20)	45.4% (54)
	Control	5.9% (7)	29.4% (35)	64.7% (77)

Table 4 reports a notable difference in the mathematical reasoning skills between both groups, with 54.6% of the experimental group demonstrating a considerable level of the reasoning skill, relative to only 35.3% of the control group (HL combined with ML). The experimental group recorded a higher percentage of participants in the HL category (37.8% versus 5.9%) – an indication that a larger portion of the group properly interpreted and translated the questions into solvable forms. Noteworthy again is that a greater percentage of participants of the control group was deficient in this aspect (64.7% versus 45.4%). This suggests that the experimental group had a stronger grasp of the reasoning skill being measured here. These findings have clarified sub-research question 2.

3. Connection of Current Mathematical Problems to Prior Knowledge

Constructing new knowledge becomes more attainable when students successfully integrate their prior learning with the current content, enabling a re-evaluation of their perceptions and an adjustment of their understanding. This is critical in subjects like mathematics, where concepts build sequentially, as integrating prior knowledge with new content facilitates deeper understanding and perspective shifts (Brod, 2021; Geofrey, 2021; Schumacher & Stern, 2023). This study investigated participants' ability to connect foundational skills – such as factorization, graph interpretation, differentiation, and geometric calculations – to the concept of stationary points in differential calculus. Post-test analysis, particularly for the experimental group, revealed varying degrees of success in leveraging prior learning, pointing to the role of prerequisite skills in clarifying challenging mathematics topics and their potential for refining instructional strategies.

Sub-RQ 3: *What level of mathematical reasoning do the students demonstrate in connecting questions to their prior knowledge as a clue to the required solutions?*

Table 5: Level of Participants' Skillful Connection of Problems to Prior Knowledge

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	34.5% (41)	17.6% (21)	47.9% (57)
	Control	1.7% (2)	30.1% (36)	68.1% (81)

As unveiled by Table 5, the percentage of high problem solvers (34.5%) in the experimental group was about twenty times higher than the 1.7% in the control group. Also, for every three low problem solvers in the experimental group, there were four in the control groups, a substantial difference of twenty-four more low problem solvers in the control group. Although the number of medium problem solvers in the experimental group here is about half of that of the control group, the experimental group still generally outperformed the control group in this achievement category. By this, sub-research question 3 is answered.

4. Application of Correct Formulas and Principles

The five post-test questions were designed to assess participants' use of skills in identifying and applying the applicable formulas and underlying principles to arrive at the correct solutions. For instance, questions 1.1, 2.3, 3.3 and 4.3 necessitated the recall of the principle $f^{'}(x) = 0$, as well as the understanding of the principles of factorization and substitution, to determine the coordinates of the turning points of the function of f and the equation of the tangent to the graph. Question 1.2 called for the use of both the principles of factorization and the general method of solving a quadratic equation. Questions 1.3, 2.4 and 3.4 involved the fundamental rules of drawing, identifying and labelling points on a graph. Questions 1.4 and 3.5 demanded recalling the basic principle $f^{''}(x) > 0$ for a graph to be concave up. The y-intercept and x-intercept of the function of f were respectively determined in questions 2.1 and 2.2 by applying the basic intercept principle of x = 0 at y-intercept and y = 0 at x-intercept. For questions 4.5 and 5.1, the participants' understanding of the rule of inflection $y^{''} = f''(x) = 0$ was necessary. Table 4 presents the analysis of the 119 post-test scripts from the experimental group and the 119 post-test scripts from the control group, based on how the participants correctly identified and applied these important formulas and principles.

Sub-RQ 4: *What level of mathematical reasoning do the students demonstrate in applying correct formulas and principles to solve questions?*

Table 6: Level of Participants' Skillful Application of Correct Formulas and Principles

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	36.1% (43)	14.3% (17)	49.6% (59)
	Control	4.2% (5)	24.3% (29)	71.4% (85)

The data from Table 6 indicates that over one-third (43 out of 119) of the experimental group participants were rated as having a high level (HL) of mastery in correctly identifying and applying the required formulas and principles to solve the given questions. An additional 17 participants (14.3%) were at a medium level (ML), showing progress towards this mastery. Although 59 participants (LL = 49.6%) were depicted as being at the low level (LL) of this skill, a majority of the group (60 out of 119, or 50%) demonstrated a considerable performance in this aspect, with HL and

ML combined. In sharp contrast, the control group exhibited a very different situation of about three-quarters (85 out of 119, or 71.4%) were rated LL, being unable to appropriately identify and apply necessary formulas and principles to tackle the questions. This suggests that the experimental group, taught with the 8Ps instructional method, demonstrated a greater level of reasoning skills in finding and applying proper formulas and principles, compared to the control group which received traditional lessons. Thus, sub-research question 4 is answered.

5. Demonstration of Logical and Sequential Solution-steps

Much like the 8Ps learning model, every heuristic mathematical problem-solving framework recognizes the essentiality of logically and sequentially presenting the solution-steps. The logical progression of the solution steps is a fundamental element of a heuristic framework which enables the problem solver to work through a task in a clear and organized manner (Favier & Dorier, 2024; Kaitera & Harmoinen, 2022; Wakhata et al., 2023). Consequently, appraising the participants' skills in this regard became imperative.

Sub-RQ 5: *What level of mathematical reasoning do the students demonstrate in showing logical and sequential solution-steps for assigned questions?*

Table 7: Level of Participants' Skillful Demonstration of Logical and Sequential Solution-steps

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	34.5% (41)	24.5% (23)	46.2% (55)
	Control	0% (0)	26.9% (32)	73.1% (87)

In Table 7, about one-third of the post-test scripts from the experimental group (34.5%) exhibited high skills in providing the logical and sequential problem-solving solution-steps that produced their solutions. Conversely, none from the control group achieved this level. Similarly, a significantly larger percentage (73.1%) of post-test scripts from the control group were categorized as low-level problem solvers as against 46.2% of the experimental group. Even though the control group had a slightly higher proportion of medium users of this skill, the experimental group was shown to be better overall. This responds to sub-research question 5.

6. Reasonable Justifications Provided for Solution-steps

The process of solving a mathematical problem requires providing a justifiable reason for each solution strategy applied (Favier & Dorier, 2024; Wakhata et al., 2023). In tune with this stance, the post-test scripts were evaluated in this direction as well.

Sub-RQ 6: *What level of mathematical reasoning do the students demonstrate in providing appropriate justifications for their solution-steps?*

Table 8: Level of Participants' Reasonable Justifications Provided for Solution-steps

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	37% (44)	16.8% (20)	41.2 % (55)
	Control	0% (0)	37% (44)	63% (75)

Table 8 displays the high- and medium-level results of the experimental group accordingly as 37% and 16.8%, portraying their ability to reasonably explain how they solved the problems. The control group, on the contrary, was only able to obtain a medium accomplishment of 37%, and no high achievement. The ratio 5:7 for both groups' low-achievement ratings is equally favorable to the experimental group. This was another area, on average, where the experimental group was stronger. Therefore, the sixth sub-research question is addressed.

7. Ability to Explain and Apply the Solutions Obtained

Most heuristic problem-solving models consider that, after determining the solution to a mathematical problem, a problem solver is expected to explain and apply the solution to related and real-life problems (Favier & Dorier, 2024; Wakhata et al., 2023). Particularly as it applies to novice mathematics students who produced the test-scripts being analyzed in this study, we only looked for evidence that relevant clues were tapped from some of the questions or solutions to understand and answer certain related questions of the post-test. For example, question 2.5 [*write down the values for which* $f''(x) < 0$] which centres on downward concavity may serve as a useful hint for solving question 1.4 [*For which values of x will the graph be concave up?*], and vice versa. Helpful solution strategies for tackling question 4.4 [*The graphs f, f' and f'' all pass through the point (0; 0).* For which of the graphs will (0; 0) be a stationary point?] can be deduced from question 3.1 which already provided another cubic graph showing the three types of stationary points.

Sub-RQ 7: *What level of mathematical reasoning do the students demonstrate in explaining and applying their solutions to understand related questions?*

Table 9: Level of Participants' Abilities to Explain and Apply the Solutions Obtained

Object of Analysis	Group	High Level (HL)	Medium Level (ML)	Low Level (LL)
Post-test scripts	Experimental	34.5% (41)	17.6% (21)	50.4% (60)
	Control	0% (0)	31.9% (38)	68.9% (81)

Table 9 discloses that the experimental group attained 34.5% high-achievement level, while the control group was unable to secure any high achievement. The table further notes that, when juxtaposed with the control group, the experimental group recorded fewer low achievers. This is evidenced by the significant difference of 21 (18.5%) in their numbers of low achievers. Overall, the experimental group emerged superior to the control group in terms of skillfully explaining and applying the solutions, notwithstanding that the control group gathered twice as many medium achievers as the experimental group. This addresses sub-research question 7.

DISCUSSION

This study's results found that, across the seven MPSAF benchmarks, the experimental group's post-test responses displayed stronger mathematical reasoning and more efficient use of the basic skills in solving the five questions on stationary points within differential calculus. With a better accuracy than the control group did, the experimental group used relevant formulas and principles, interpreted and translated the given mathematical problems into solvable forms and represented the problems in useful patterns. Furthermore, the experimental group depicted greater skills in incorporating prior knowledge into current learning, provided logical and orderly solution-steps with verifiable reasons, and applied their solutions to enhance their understanding of related problems. The finding of MPSAF criterion 1 aligns with Tirpáková et al. (2023) who observe that task substitution and transformation strategies encourage students to develop their own approaches to algebraic problems, thereby improving their mathematical problem-solving skills in interpreting and transforming complex mathematical tasks into solvable forms.

The finding under MPSAF criterion 2 is in line with Kusumaningsih et al. (2018) who obtain that multiple representation strategies in Realistic Mathematics Education (RME) enhanced students' ability to reformulate algebraic tasks into diverse helpful mathematical representations (such as equations, graphs, charts, tables, etc.), causing the experimental group taught through this approach to outperform the control group given traditional instruction. Mirzaei (2024) supports the finding from MPSAF criterion 3 by confirming that linking differential calculus concepts to prior knowledge strengthens reasoning and solution strategies. Maciejewski and Star (2016) showed that students trained in flexible procedural knowledge exhibited considerable improvements in applying mathematical concepts and procedures to solve differential calculus problems, thus corroborating the MPSAF criterion 4.

The findings got by criteria 5 and 6 of the MPSAF resonate with Favier and Dorier (2024) who highlight that using heuristics effectively improves students' logical reasoning and ability to structure and clarify solution steps properly. Lastly, Wakhata et al. (2023) demonstrates that for further acceptability of the solution, problem solving should extend beyond obtaining the solution to exploring its application in solving related mathematical and real-world problems. The foregoing indicates that this study has answered the seven sub-research questions, capturing the reasoning that informed participants' solution approaches and their skill areas needing further development, such as formula selection, expansion of cubic expressions, graphical application of differential calculus knowledge, and interpreting concavity within differential calculus. Even though the experimental group showed some improvement, both groups require additional support in these mathematical areas.

CONCLUSION AND RECOMMENDATIONS

This study employed a document-analysis technique to probe the influence of the 8Ps learning model on how South African grade 12 mathematics students solve problems in the domain of stationary points in differential calculus. The findings corroborate and consolidate on our previous research, suggesting that the 8Ps learning model has the capacity to enhance students' mathematical problem-solving skills. Consequently, the study recommends the adoption of this model in the mathematics classroom. It is important to acknowledge that the 8Ps model is relatively recent, and this study has some limitations, such as a restricted scope, short duration, small sample size and non-randomized participant selection. As such, further research is recommended for a more comprehensive evaluation of the model's potential to improve mathematics instruction by implementing it for more mathematics concepts in different grade levels, subject areas, educational and socio-economic contexts making use of various relevant research methods.

THE STUDY'S CONTRIBUTION TO SCHOLARSHIP

- This study highlights the value of document analysis within qualitative research for assessing students' academic performance, advocating for its regular use like other qualitative research methods.
- The study fosters a structured process (through its seven-parameter MPSAF) for analyzing how students solve mathematical problems, offering practical strategies for measuring students' progress and difficulties in mathematical problem solving.
- The study employs the cognitive principles of John Dewey and Graham Wallas to analyze students' mathematical problem-solving performance, promoting the relevance of both theories individually and conjointly, while also establishing a theoretical basis for the development and relevance of the 8Ps learning model.
- Additionally, the study provides empirical evidence triangulating the earlier findings which suggest that the 8Ps learning model possesses the potential to improve students' mathematical problem-solving skills. These insightful findings can help shape future instructional practices, contributing to improved teaching and learning experiences in differential calculus and broader secondary mathematics education.

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Appendix A1

Mathematical Problem-solving Assessment Form (MPSAF)

Assessment Criteria ***Participants' ability to:***	High Level (HL)	Medium Level (ML)	Low Level (LL)	Total
Represent mathematical problems as helpful patterns				
Translate mathematical problems to solvable forms				
Connect current mathematical problems to previous knowledge				
Apply proper formulas and principles to solve given questions				
Demonstrate logical and sequential solution-steps				
Provide reasonable justifications for solution-steps				
Explain and apply the solutions obtained to related problems				
Total				

Appendix A2

Instruction: Answer all the questions showing clearly all your calculations.

Duration: $1\frac{1}{2}$ hours

QUESTION 1 (Feb-March 2017 Q8)

Given: $f(x) = 2x^3 - 5x^2 + 4x$.

1.1 Calculate the coordinates of the turning points of the graph of f. (5)

1.2 Prove that the equation $f(x) = 2x^3 - 5x^2 + 4x$ has only one root. (3)

1.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

1.4 For which values of x will the graph be concave up? (3)

[14]

QUESTION 2 (May-June 2017 Q9)

Given: $f(x) = x^3 - x^2 - x + 1$.

2.1 Write down the coordinates of the y-intercepts of f. (1)

2.2 Calculate the coordinates of the x-intercepts of f. (5)

2.3 Calculate the coordinates of the turning points of f. (6)

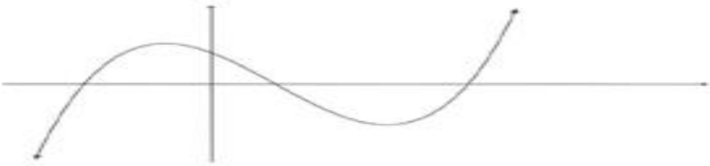
2.4 Sketch the graph of f. Clearly indicate all intercepts with the axes and the turning points. (3)

2.5 Write down the values for which $f''(x) < 0$. (2)

[17]

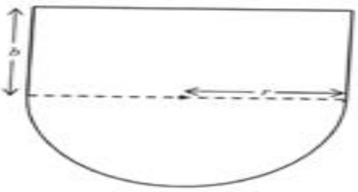
QUESTION 3 (Feb-March 2018 Q9)

The sketch below represents the curve of $f(x) = x^3 - bx^2 + cx + d$.
The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4.



3.1	Calculate the values of b, c and d.	(4)
3.2	Calculate the coordinates of B, the maximum turning point of f.	(4)
3.3	Determine an equation for the tangent to the graph of f at $x = 1$.	(4)
3.4	Sketch the graph of $f''(x)$ and clearly indicate the x- and y-intercepts.	(3)
3.5	For which value(s) of x is $f(x)$ concave upwards?	(2)
		[17]

QUESTION 4 (May-June 2017 Q10)



The figure is a design of a theatre stage in the shape of a semi-circle attached to a rectangle. The semi-circle has a radius r and the rectangle has a breadth b. The perimeter of the stage is $60\ m$.

4.1	Determine the expression for b in terms of r.	(2)
4.2	For which value(s) of r will the area of the stage be a maximum?	(6)

Given that $f(x) = 3x^3$. (Nov 2019 Q9)

4.3	Solve $f(x) = f'(x)$.	(3)
4.4	The graphs f, f' and f'' all pass through the point (0 ; 0). For which of the graphs will (0 ; 0) be a stationary point?	(1)
4.5	Explain the difference, if any, in the stationary points mentioned in (4.4).	(2)
4.6	Determine the vertical distance between the graphs of f' and f'' at $x = 1$.	(3)
		[17]

QUESTION 5

5.1 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (May-June 2017 Q8.3) (7)

5.2 Given: $f(x) = -3x^3 + x$. Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. (Feb-March 2018 Q10) (6)

5.3 A piece of wire $6\ m$ long is cut into two pieces. One piece, $x\ m$ long, is bent to form a square ABCD. The other piece is bent into a U-shape to form a rectangle BEFC when placed next to the square.

Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. (Feb-March 2017 Q9) (7)

[20]
TOTAL: 85